Additional Design Procedures

Procedure for Analysis of Pump Tower and Pump Tower Base

September 2008
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Chapter 1: Introduction and Applicability

Section 1. Introduction

The strength, Fatigue and Vibration analysis of the Pump Tower and Pump Tower Base Support (PTBS) of Membrane Tank LNG ships is mandatory.

This procedure provides guidelines for the strength and fatigue analysis of Pump Tower and Pump Tower Base Support due to loads induced by ship motions, including sloshing of LNG in partially filled cargo tanks, and thermal effects. Guidelines regarding vibration analysis with the objective of avoiding resonance with propeller and, where relevant, main machinery exciting frequencies are also given.

In general, the assessment is to be based on a three-dimensional finite element analysis (3D-FEA) carried out in accordance with the procedures contained in these guidance notes.

Ships which have novel features or unusual hull structural or tank configurations or have non standard operational requirements, such as short duration voyages, frequent filling/emptying operation, off-shore unloading, will need special consideration.

It is recommended that the designer consults Lloyd’s Register on the analysis requirements early on in the design cycle.

Where alternative procedures are proposed, these are to be agreed with Lloyd’s Register before commencement.

If equivalent software is employed, full particulars of the software with its validation procedure may be required to be submitted.

Lloyd’s Register may require the submission of computer input and output to further verify the adequacy of any of the calculations carried out.

Section 2. Symbols

The symbols used in these guidance notes are as follows:

- \( L \) = rule length, as defined in Pt 3, Ch 1, 6 of the Rules for Ships, m
- \( B \) = moulded breadth, as defined in Pt 3, Ch 1, 6 of the Rules for Ships, m
- \( GM \) = transverse metacentric height with free surface correction, m
- \( L_T \) = tank length (inside containment system insulation), m
- \( H \) = tank depth (inside containment system insulation, m
- \( V_T \) = tank volume, m³
- \( F \) = cargo fill height in the tank, m
Section 3: Applicability

This procedure is applicable for the strength and fatigue analysis of the Pump Tower and Pump Tower Base Support fitted to membrane tank LNG ships with cargo tank proportions and dimensions complying with containment system designer’s recommendations. It is assumed that the Pump Tower is located, in accordance with normal practice, near the aft end of the tank close to the centreline of the ship, as shown in Figure 1.1.

Application of the procedure is restricted to membrane tank LNG ships intended for operation with an assigned barred fill range. Hence, it is applicable for the following high and low filling ranges:

(i) Below the maximum allowable low fill level. Typically this is 10%H but may be lower. This value is to be taken as the agreed highest allowable fill height below the barred fill range applicable for the ship.

(ii) Above the minimum allowable high fill level. Typically this is 70%H or higher. This value is to be taken as the agreed highest allowable fill height above the barred fill range applicable for the ship.

The strength analysis of the Pump Tower with respect to the loadings imposed during cargo pumping operations is not covered by this procedure and is to be submitted as a separate document. For ships which load and unload in sheltered conditions, the loads arising from cargo pumping operations need not be combined with those loading components described in this procedure.

The double sleeve arrangements of pipes which pass through the inner trunk deck to guard against thermal stresses are not covered by this procedure and are to be considered separately.

Where the design or location of the Pump Tower, or the intended cargo tank filling levels do not comply with the above, then the Pump Tower arrangement is to be specially considered using this procedure as a basis.
Section 4: Analysis Report

A report containing details of the strength, fatigue and vibration analyses undertaken for the Pump Tower and Pump Tower Base Support is to be submitted. This is to include:

- a list of plans used, including dates and versions
- the methodologies used for analysing the pump tower and pump tower base support
- details of the ship motion codes and their validation or, model tests used to compute ship motions in irregular seas, if alternative methods are used
- details and validation of the Computational Fluid Dynamics (CFD) codes and/or the model test procedures used to predict the dynamic loads due to sloshing of LNG
- details of applied loadings including hydrodynamic forces and inertial forces of LNG in partially filled cargo tanks, thermal effects and confirmation that individual and total applied loads are correct
- a detailed description of the structural model, including all modelling assumptions, material properties and boundary conditions;
- plots and results to demonstrate that the behaviour of the structural model to the applied loads is correct;

The analysis report is to include the maximum allowable deflections of the Pump Tower Base Support as specified by the designer of LNG containment system.
Chapter 2: Summary of Procedure

Section 1: General

The main steps involved in verifying the strength, fatigue and vibration characteristics of the Pump Tower and Pump Tower Base Support, for each selected combination of tank fill height and critical loading condition, are given below:

- Evaluation of the velocity of the LNG flowing past the Pump Tower due to sloshing caused by ship motions (see Lloyd's Register's ShipRight SDA Procedure for Sloshing Loads and Scantling Assessment) using LRFLUIDS, suitable CFD software or suitable model tests.

- Calculation of the forces and moments resulting from the flow of LNG past the Pump Tower using Morison's formula.

- Structural analysis of the Pump Tower and PTBS to evaluate the deflections and stresses arising from the flow of LNG, in combination with inertial loads (including self weight of steel structure and entrapped fluid in the pipes) due to ship motion accelerations.

- Structural analysis of the Pump Tower to calculate the stresses induced by the temperature distribution over the height of the Pump Tower and the temperature difference between the Pump Tower and the supporting structure.

- Evaluation of the results for the possible failure modes, i.e. yield strength, punching strength, buckling strength and fatigue strength.

- Evaluation of the natural frequency of the pump tower to ensure that it does not coincide with main machinery or propeller blade excitation frequencies (resonance avoidance analysis).
Chapter 3: Estimation of Fluid forces

Section 1. Pre-Calculation and Modelling
Section 2. Loading Conditions
Section 3. Hydrodynamic Forces due to Flow of LNG

Section 1: Pre-Calculation and Modelling

The ship’s loading manual is to be reviewed to find the seagoing loading conditions which give the greatest GM value. For ballast loading conditions, the selected GM is to be the maximum from all the conditions in which the filling level in all cargo tanks is below the allowable low fill height. For loaded loading conditions, the selected GM is to be the maximum from all conditions in the loading manual with at least one full cargo tank. This is often found to be the loading condition with No. 1 tank full and Nos. 2, 3 and 4 tanks empty.

The procedure is based on the use of Lloyd’s Register's ShipRight SDA Sloshing procedure and supporting software (the LRFLUIDS CFD program). Alternative software or modelling techniques may be used, but the proposed procedure and validation of the CFD software used is to be agreed with Lloyd's Register prior to commencement of the analysis.

ShipRight SDA Level 1 Sloshing calculations or equivalent are to be carried out to find the ship natural roll period and the tank natural roll periods for a range of filling levels using a midship cargo tank.

A model of the transverse section of the largest midship cargo tank is to be prepared using LRFLUIDS (ShipRight SDA Level 3 Sloshing software) or an alternative acceptable CFD sloshing program. The following properties are to be assumed for the liquid LNG:

- Density \( \rho = 470 \text{ kg/m}^3 \)
- Speed of sound = 1366 m/s
- Kinematic viscosity = 2.5*10^{-7} m^2/s

Section 2: Loading Conditions

The LNG fluid forces acting on the Pump Tower due to sloshing are to be calculated for two ship rolling conditions as follows:

1. fluid forces due to ship motions coincident with the ship natural roll period
2. fluid forces due to ship motions coincident with the tank natural roll period.

For ship configurations in which the pump tower is mounted close to a transverse bulkhead, it is not necessary to consider a pitching case.

2.1 Ship Natural Roll Period Case

The LNG fluid motions are to be calculated for the following fill heights and loading conditions for ship motions which are coincident with the ship natural roll period:

1. Ship natural roll period based on the GM selected from the review of ballast loading conditions:
   - Maximum allowable low fill height, usually 10% H, see Ch 1, 3

2. Ship natural roll period based on the GM selected from the review of loaded loading conditions
   - Minimum allowable high fill height, usually 70%H, see Ch 1, 3
   - 95% H
Intermediate fill heights as considered necessary (say F=80%H and F=90%H)

For all these fill height conditions, the roll angle is to be entered as 70% of the lifetime roll angle and the heave and sway values are to be entered as 100% of the lifetime values. The maximum lifetime ship motions are to be calculated as given below (see ShipRight SDA procedure for Sloshing Loads and Scanlcing Assessment):

\[
\begin{align*}
&\text{Roll angle, } \Phi_{\max} = (14.8 + 3.7L/B)e^{-0.0023L} \text{ deg} \\
&\text{Heave, } Z_{\max} = 10e^{-0.0032L}, \text{ but not greater than } 4.0 \text{ metres} \\
&\text{Sway, } Y_{\max} = 5e^{-0.0025L}, \text{ but not greater than } 2.5 \text{ metres}
\end{align*}
\]

The ship natural roll period is to be taken as:

\[
\text{Ship natural roll period } = S_{\text{nr}} = 2.35 \frac{r}{\sqrt{GM}}
\]

where:
- \(r\) is the radius of gyration of roll which may be taken as 0.34 B, m
- \(GM\) is the transverse metacentric height with free surface correction, m

The LNG fluid forces for each of the filling heights are to be assessed by considering a range of excitation periods around the ship natural roll period of at least +/-2 seconds.

For high fill levels, it will usually be found that the fluid motion is of a swirling (rotating) nature such that the flow near the LNG surface is in the opposite direction to that near the bottom of the tank (Figure 3.1). This flow pattern reverses in direction and then repeats itself in response to the motions of the ship.

The derivation of fluid forces acting on the Pump Tower is based on the maximum fluid velocity acting anywhere along the Pump Tower height, see Section 3. Hence, for each filling level, the time series of fluid velocities needs to be obtained and the maximum value at each cell location identified.

Figure 3.1: Swirling type fluid motion in the Tank
2.2 Tank Natural Roll Period Case

The LNG fluid motions are to be calculated for the following fill heights and loading conditions for ship motions coincident with the tank natural roll period. The tank natural roll period is to be determined as given in the ShipRight SDA Level 1 procedure, or equivalent:

1. Tank natural roll period based on the maximum allowable low fill height, usually 10% H, see Ch 1, 3. The ballast loading condition applies.

2. Tank natural roll period based on the following:
   - Minimum allowable high fill height, usually 70%H, see Ch 1, 3
   - 95% H
   - Intermediate fill heights as considered necessary (say F=80%H and F=90%H)
   The loaded loading condition applies.

Typically the only tank natural period case that needs to be considered corresponds to minimum allowable high fill height. This case is often found to give the least difference between the tank and ship natural periods due to the chamfered shape of the tank cross-section.

The ship motions specified in 2.1 are to be applied. Alternatively, ship motion data from a ship motion program or model experiments corresponding to the appropriate loading condition (see Section 1) may be used to identify possible reductions in roll, heave and sway amplitudes, in representative wave conditions, which will cause the ship to roll at a period equal to the tank natural roll period at this fill height.

Sloshing responses for the minimum allowable high fill height are to be assessed by considering a range of excitation periods around the tank natural roll period of at least +/-2 seconds.

For this case, it is usually found that the fluid motion is in the form of a standing wave as shown in Figure 3.2, where the fluid flow near the free surface and at the bottom of the tank is in the same direction, with maximum flow velocity occurring at the free surface. The maximum loading on the Pump Tower is based on the maximum fluid velocity acting anywhere along the Pump Tower height. Hence, for each filling level, the time series of fluid velocities needs to be obtained and the maximum value at each cell location identified.

![Figure 3.2: Standing wave type motion in the Tank](image-url)
Section 3: Hydrodynamic Forces due to Flow of LNG

The calculation of hydrodynamic forces due to the flow of LNG fluid past the Pump Tower is to be performed for:

- the discharge pipes,
- the emergency pump guide pipe (the forward vertical pipe in the tripod arrangement),
- the cross-bracing between the discharge pipes and the emergency pump guide pipe,
- any other pipes of significant size exposed to the flow.

The following items may be neglected when calculating the hydrodynamic forces on the pump tower for the ship and tank natural roll cases:

- the filling pipes
- cross-bracing between the two discharge pipes, provided that the two discharge pipes (aft vertical pipes in the tripod arrangement) are located symmetrically about a plane perpendicular to the ship centreline. In the roll situation, these discharge pipes are considered to shield the cross bracing between them from the effects of the flow of LNG.

The hydrodynamic force, $F$, on each Pump Tower pipe segment or element exposed to the flow of LNG is to be calculated from the following formula. This calculation is based on Morison's equation but is modified to account for the fluid velocity and fluid acceleration forces, $F_U$ and $F_A$, (as calculated in this procedure) being taken as 90 degrees out of phase

$$F = (F_U^2 + F_A^2)^{1/2} \text{ N}$$

where:

- Force due to fluid velocity: $F_U = 0.5 \, C_d \, \rho \, D \, U \, |U| \, dl \text{ N}$
- Force due to fluid acceleration: $F_A = C_m \, \rho \, A \, U' \, dl \text{ N}$

$C_d$ = equivalent drag coefficient, to be taken as 0.7
$\rho$ = density of LNG, see Section 1, in kg/m$^3$
$D$ = outside diameter of pipe, in m
$U$ = maximum fluid velocity acting on each pipe segment, dl, along the height of the Pump Tower location, in m/s
$|U|$ = absolute value of U, in m/s$^2$ acting on each pipe segment
$U'$ = maximum acceleration of LNG acting on each segment, dl, to be taken as $2\pi \, U / T$, in m/s$^2$
$T$ = ship natural roll period or tank natural roll period, as appropriate, in sec
$dl$ = length of the pipe segment or element between braces, etc, in m
$A$ = cross sectional area of pipe, $= \pi D^2/4$, in m$^2$
$C_m$ = geometric inertia coefficient, to be taken as 2.0 for tubular pipes

In applying this procedure, it should be noted that four load cases are required for each filling level (see Ch 4, 3). These load cases apply the four possible combinations of the maximum positive and maximum negative envelope hydrodynamic fluid forces and maximum positive and maximum negative ship motion inertial forces (see Ch 4, 2.2) to ensure that the maximum load that could be applied to the Pump Tower is covered.

The two possible scenarios regarding hydrodynamic fluid forces are:

1. forces derived using the maximum positive values of fluid velocity, $U$.
2. forces derived using the maximum negative values of fluid velocity, $U$.

The two possible scenarios regarding inertial forces (see Ch 4, 2.2) are:

1. forces derived using the maximum ship motion positive acceleration value acting on any PT
2. forces derived using the maximum ship motion negative acceleration value acting on any PT

For the maximum positive fluid velocity case, the procedure is as follows:

- For each Pump Tower pipe element, the maximum positive value of fluid velocity, $U_+$, along its length as predicted by the CFD analysis is to be established
For each Pump Tower pipe element the corresponding acceleration term, $U_+^*$, is to be derived from the predicted velocity using the equation

$$U_+^* = \frac{2\pi U_+}{T}$$

For each Pump Tower pipe element, the force acting on the element, $F_+$, is calculated from

$$F_+ = \left( F_{+U}^2 + F_{+A}^2 \right)^{\frac{1}{2}}$$

The maximum force, $F_+$, for each pipe element is to be applied simultaneously to the FE model irrespective of the phase relationship between the fluid velocities acting on individual pipe segments

For the maximum negative fluid velocity case, the procedure is as follows:

- For each Pump Tower pipe element, the maximum negative value of fluid velocity, $U_-$, along its length as predicted by the CFD analysis is to be established
- For each Pump Tower pipe element the corresponding acceleration term, $U_-^*$, is to be derived from the predicted velocity using the equation

$$U_-^* = \frac{2\pi U_-}{T}$$

- For each Pump Tower pipe element, the force acting on the element, $F_-$ is calculated from

$$F_- = \left( F_{-U}^2 + F_{-A}^2 \right)^{\frac{1}{2}}$$

- The maximum force, $F_-$, for each pipe element is to be applied simultaneously to the FE model irrespective of the phase relationship between the fluid velocities acting on individual pipe segments

In the above expressions, the subscripts $+U$ and $-U$, denote the positive and negative velocities and the subscripts $+A$ and $-A$ denote the accelerations derived from positive and negative velocities respectively.

As an alternative to using Morison's equation and the above procedure, the hydrodynamic forces can be derived using suitable CFD software to analyse a three dimensional model of the tank with the Pump Tower included. The proposed procedure and validation of the CFD software is to be agreed with Lloyd's Register prior to commencement of the analysis.
Chapter 4: Structural Analysis

Section 1. Modelling

1.1 General

Two Finite Element (FE) models are required:

1) The Pump Tower and its upper and lower support structures
2) The Pump Tower Base Support and the supporting structure in the double bottom.

Both models may be combined into one FE analysis if preferred.

The Pump Tower model is to include appropriate modelling at the top and bottom of the tower in all degrees of freedom to reflect the connection with the adjacent primary structure. Particular attention is to be given to ensuring that the correct representation of connections between Pump Tower structure and the Pump Tower Base Support, and the inner and outer trunk decks, is achieved. This is especially important for the thermal load cases.

1.2 Pump Tower (PT) Model

For the Pump Tower itself, it is sufficient to model only the main tubular columns and the cross-bracing.

In principle, a coarse mesh comprising one line element (bar element with axial shear and bending stiffness) between nodal joints is acceptable, e.g. for the main tubular column, one element between cross braces is acceptable.

However, it should be noted that stress results are required at intermediate points between the cross brace joints. Hence, if the analysis code being used is not capable of providing stress results at a minimum of seven points along the length of the element, i.e., at both ends and at 0.1 Lp, 0.25 Lp, 0.50 Lp, 0.75 Lp and 0.9 Lp (where Lp is the effective cross beam length, see Ch 5, 1.1), then six elements are required between cross brace joints. Cross brace nodal positions are to be modelled at the intersection of the centre lines of the intersecting members.

Property entries for the line elements are to include the cross-sectional area of the tube, the area moments of inertia and the torsional constant.

For cross braces with sliding ends, the relevant degree of freedom is to be released to permit the sliding motion (for example, if the analysis code is MSC NASTRAN, this can be achieved using the pin flags option).

The lower ends of the main tubular columns are connected to a base plate and, via a guide assembly, to the Pump Tower Base Support (PTBS) structure. This partially restricts the movement of the Pump Tower in the longitudinal and transverse directions. The base plate is to be modelled using a suitable arrangement of plate and line elements.

The line elements representing the tubular columns are to be connected to the base plate by suitable rigid elements (in MSC NASTRAN RBE1 or RBE2 elements are suitable) (see Figs 4.1 and 4.2). Care is to be taken to ensure that sufficient dependencies are defined so that the relevant forces and moments are transferred from the base plate to the tubular columns and that over constraint is not introduced.
Figure 4.1: Pump Tower (PT) Model (No96 design illustrated)
For the temperature case it is necessary to ensure that the selected rigid element can represent the effects of thermal contraction. (In MSC NASTRAN, the thermal expansion coefficient is entered into either the RBE1 or RBE2 definition and RIGID= LARGAN is included in the Case Control Deck)

The Pump Tower Top Support (PTTS) is to be represented as given in 1.3.

The connection to the Pump Tower Base Support (PTBS) is to be represented as given in 1.4.

Other structural items, such as platforms and large equipment items, e.g. pumps and other accessories, are to be modelled as lump masses. Minor pipes, ladders, platforms and cableways etc. may be incorporated into the model by adjusting the density of the material. Alternatively, non structural mass on the appropriate elements may be specified.

The mass of the Pump Tower model and location of its centre of gravity are to be checked against the supplied information. Adjustments to the modelled mass density and lump masses are to be made until suitable correspondence is achieved.
1.3 Pump Tower Top Support (PTTS)

As indicated in Chapter 1, 3, the assessment of the double sleeve and insulation arrangements of the main tubular columns through the inner trunk deck, provided to guard against thermal stresses, does not form part of these guidelines. However, appropriate representation of the Pump Tower Top Support (PTTS) is required to ensure that the correct response of the pump tower to the applied loads is obtained.

The PTTS arrangements depend on the type of containment system fitted and the exact method of modelling them will depend on the proposed support arrangements. Typically, the following methods are recommended for the GTT Pump Tower Top Support Systems.

For containment systems without a large opening in way of the pump tower and in which the PTTS connects to the inner trunk deck, such as in the GTT No96 design, the upper connections of the main vertical tubular columns to the inner and outer trunk deck may simply be represented by boundary conditions as follows:

- Gap elements in the transverse and longitudinal directions and arranged at the inner deck level can be used to represent the thermal sleeve arrangement. The gap is to be equal to the clearance between the inner and outer sleeves. For pump towers which have a triangular arrangement in plan view and, where two of the main columns lie in the transverse plane, these gap elements need only be applied to these two columns. The third column is to be constrained in both the transverse and longitudinal directions, see Fig 4.1
- Vertical constraints at the inner deck level.
- All other translations and rotations at the inner trunk deck and outer trunk deck level are to be free of constraint.

For containment systems having a large liquid dome opening in the trunk deck in way of the pump tower location such as GTT MK III design, the following is to be applied, see example in Fig 4.3:

- Plate and line elements are to be used to represent the hatch cover at the outer trunk deck level. It is not necessary to include the dome opening coaming.
- The hatch cover is to be constrained in all translations and rotations at its boundaries
- For pump towers which have a triangular arrangement in plan view and where two of the main columns lie in the transverse plane, the connection between the members representing the pipe tower columns and those representing the stiffening of the hatch cover are to be removed in the longitudinal and transverse directions (e.g. by use of MSC NASTRAN pin flags or an equivalent facility) and replaced by gap elements. The gap is to be equal to the clearance between the inner and outer sleeves.

1.4 Connection between Pump Tower and Pump Tower Base Support.

The Pump Tower is connected, via a guide assembly which partially restricts the movement of the Pump Tower in the longitudinal and transverse directions, to the Pump Tower Base Support (PTBS) structure.

The sliding connection provided by the guide assembly is to be represented by a suitable arrangement of gap or sliding elements in way of the interface between the Pump Tower base plate and the Pump Tower Base Support (see Fig 4.4).

This applies whether the PTBS model is included in the PT analysis or if the PTBS model is analysed separately. Normally, the guide assembly location on the Pump Tower Base plate should be constrained so as to prevent rotation of the Tower about its vertical axis and translation of the Tower in the longitudinal and transverse directions. The vertical translation and other rotational directions are to be free of constraint.
Procedure for Analysis of Pump Tower and Pump Tower Base, September 2008

Chapter 4

1.5 Pump Tower Base Support (PTBS) Model

Plate and bar elements are to be used to represent the Pump Tower Base Support and the double bottom, including the local reinforcement structure in way of the PTBS. The double bottom model is to extend at least to the 1st transverse and 1st girder outside the extent of the local reinforcement.

The mesh size is to be such that openings in the PTBS and double bottom structure are reasonably well represented in the model and the deflection response of the PTBS is correctly predicted. The mesh size is not to exceed about 50 mm x 50 mm for the stress analysis but see also 5.6 which requires a $t \times t$ mesh for the fatigue check in way of fatigue critical regions.

The Pump Tower connection is to be modelled as indicated in 1.4 if included within the PT model. If the PTBS model is analysed separately, then the loads from the PT analysis are to be applied as forces to the PTBS model.
Figure 4.4: Arrangement of gap or sliding elements representing the guide assembly between the pump tower base plate and the Pump Tower Base Support (PTBS)
Section 2: Application of Loads

2.1 Hydrodynamic loads

The hydrodynamic forces on the Pump Tower due to sloshing, as determined from Ch 3, are to be applied as pressure or distributed loads along the length of all line elements representing pipes exposed to the fluid flow. (e.g. in MSC NASTRAN, the PLOAD1 card is a suitable load application format). The hydrostatic forces on the Pump Tower may be ignored.

2.2 Inertia Loads

The whole Pump Tower and any LNG contained within the pipes of the Pump Tower are to be subjected to inertial loading due to the transverse acceleration of the ship. The applicable transverse acceleration is to be calculated in accordance with the guidance formula in paragraph 4.12 of the IGC Code. The transverse accelerations due to ship motions are to be calculated for all Pump Towers in all cargo tanks and the maximum value acting on any Pump Tower is to be used for the inertial load calculation. These forces may to be incorporated into the model by suitable adjustment of the gravity vector applied to the model (e.g. in MSC NASTRAN, modifying the values entered on the GRAV card). Alternatively, these forces may be applied as distributed loads to all elements of the pump tower.

For the tank natural roll period case, the transverse acceleration may be reduced to a value commensurate with the reduced ship motion, see Ch 3, 2.2.

2.3 Self Weight

The self weight of the Pump Tower and its associated equipment is to be included in the strength calculations assuming a gravity vector of 1 g. The effects of buoyancy of pipes and weight of liquid in the pipes may be neglected when calculating the effective self weight of the Pump Tower.

2.4 Thermal Loads

Thermal loading is applied by defining a suitable temperature distribution over the height of the Pump Tower model. The temperature distribution over the Pump Tower height is dependent on the filling level under consideration and the voyage duration.

The following distribution is to be assumed:

For structure below fluid surface, the temperature is to be taken as -163°C.

For structure between the fluid surface and the underside of the top of the liquid dome, or the inner trunk deck for designs without a defined liquid dome in way of the Pump Tower, the temperature is to be taken as:

- for the case with fill heights of 70% H and above: -163°C
- for the case with fill heights below 10% H: -30°C

For the ship structure, ambient temperature is to be taken as +20°C at the inner trunk deck and +45°C at outer trunk deck. A linear distribution can be assumed in between.

The ambient temperature of +45°C is to be used as the reference (initial) value for the analysis.
Section 3: Load Cases

The following load cases are to be run for the strength analysis:

Load case C1n – Ship Natural Roll Period Case for the maximum allowable low fill height, usually 10% H

This load case is based on the ballast loading condition with a maximised GM (see Ch 3, 1 and Ch 3, 2.1). The load case is to comprise of the following load components:

(a) Hydrodynamic forces for the maximum allowable low fill height
(b) Transverse inertial loading for the maximum allowable low fill height
(c) Self weight due to gravity vector of 1 g
(d) Thermal loads for the maximum allowable low fill height

Load case C2n – Ship Natural Roll Period Case for the minimum allowable high fill height, usually 70% H

This load case is based on the loaded loading condition with a maximised GM (see Ch 3, 1 and Ch 3, 2.1). The load case is to comprise of the following load components:

(a) Hydrodynamic forces for the minimum allowable high fill height
(b) Transverse inertial loading for the minimum allowable high fill height
(c) Self weight due to gravity vector of 1 g
(d) Thermal loads for the minimum allowable high fill height

Load case C3n – Ship Natural Roll Period Case for a fill height of 95% H

This load case is based on the loaded loading condition with a maximised GM (see Ch 3, 1. and Ch 3, 2.1). The load case is to comprise of the following load components:

(a) Hydrodynamic forces for a fill height of 95% H
(b) Transverse inertial loading for a fill height of 95% H
(c) Self weight due to gravity vector of 1 g
(d) Thermal loads for a fill height of 95% H

Load case C4n – Tank Natural Roll Period Case for the minimum allowable high fill height, usually 70% H

This load case is based on the loaded loading condition with a maximised GM (see Ch 3, 1 and Ch 3, 2.2). The load case is to comprise of the following load components:

(a) Hydrodynamic forces for the minimum allowable high fill height
(b) Transverse inertial loading for the minimum allowable high fill height
(c) Self weight due to gravity vector of 1 g
(d) Thermal loads for the minimum allowable high fill height

Additional tank natural roll period load cases may be required to take account of any specified partial fill conditions, where the tank natural roll period is close to the ship natural period.

For each of the load cases C1n to C4n, the individual load components are to be combined as indicated in the following Table:

---

1 The subscript n in C1n to C4n is used to denote that a set of load cases (n = 1 to 4) is to be analysed for each main load case shown above.
### Load case ID

<table>
<thead>
<tr>
<th>Load component</th>
<th>Load cases C1, C2, C3, &amp; C4</th>
<th>Load cases C12, C22, C32, &amp; C42</th>
<th>Load cases C13, C23, C33, &amp; C43</th>
<th>Load cases C14, C24, C34, &amp; C44</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Hydrodynamic LNG forces</td>
<td>+ve fluid forces, $F_r$</td>
<td>+ve fluid forces $F_r$</td>
<td>-ve fluid forces $F_r$</td>
<td>-ve fluid forces $F_r$</td>
</tr>
<tr>
<td>b) Inertial forces</td>
<td>+ve inertial force</td>
<td>-ve inertial force</td>
<td>-ve inertial force</td>
<td>+ve inertial force</td>
</tr>
<tr>
<td>c) Self-weight</td>
<td>Include</td>
<td>Include</td>
<td>Include</td>
<td>Include</td>
</tr>
<tr>
<td>d) Thermal loads</td>
<td>Include</td>
<td>Include</td>
<td>Include</td>
<td>Include</td>
</tr>
</tbody>
</table>

Notes:
- Load components a), b), c) & d) are as described in preceding paragraphs
- +ve and -ve load components are defined below:
  - +ve fluid forces, $F_r$, are fluid forces in the direction of +ve velocity, as calculated using Ch 3, 3
  - -ve fluid forces, $F_r$, are fluid forces in the direction of -ve velocity, as calculated using Ch 3, 3
  - +ve inertial forces are consistent with roll to starboard
  - -ve inertial forces are consistent with roll to port

When separate models of the Pump Tower and Pump Tower Base Support are used, the load cases should first be applied to the model of the Pump Tower. The Interaction forces obtained from this analysis should then be applied to the separate model of the Pump Tower Base Support.

The load cases to be run for the fatigue analysis are given in Ch 5, 3.3
Chapter 5: Analysis of Results

Section 1. Pump Tower - Stress and Buckling Assessment

The Pump Tower tubular columns and cross braces are to satisfy the following stress and buckling criteria. At least seven positions along each pipe length between cross brace joints are to be examined, namely 0.0L_p, 0.1L_p, 0.25L_p, 0.5L_p, 0.75L_p, 0.9L_p and 1.0L_p, where L_p is defined in 1.1.

For all load cases, the axial and bending stresses and their combinations at each position, are to satisfy the assessment criteria given in 1.2.

Cross braces which are completely or partially immersed in LNG and which are fluid tight are also to satisfy the requirements of 1.3. These additional checks do not normally apply to the main vertical tubular members which are assumed to be filled with LNG up to the same level as the tank fill height.

1.1 Allowable stresses for cylindrical members

Allowable axial tensile stress, \( \sigma_t \):
\[
\sigma_t = f_a - 0.6 \sigma_y
\]

Allowable axial compressive stress, \( \sigma_c \):
\[
\sigma_c = f_a \left[ \frac{1 - (KL_p/r)^2}{2C_y^2} \right] \sigma_f
\]
\[
\text{for } (KL_p/r) < C_c
\]
\[
\sigma_c = f_a \frac{12\pi^2 E}{23(KL_p/r)^2}
\]
\[
\text{for } (KL_p/r) \geq C_c
\]

Allowable bending stress, \( \sigma_b \):
\[
\sigma_b = f_a \left[ 0.75\sigma_y \right.
\]
\[
\text{for } \left( \frac{D}{t} \leq 10340 \right)
\]
\[
\sigma_b = f_a \left[ 0.84 - 1.74 \frac{\sigma_D}{E} \right] \sigma_y
\]
\[
\text{for } \left( \frac{10340}{\sigma_y} < \frac{D}{t} \leq \frac{20680}{\sigma_y} \right)
\]
\[
\sigma_b = f_a \left[ 0.72 - 0.58 \frac{\sigma_D}{E} \right] \sigma_y
\]
\[
\text{for } \left( \frac{20680}{\sigma_y} < \frac{D}{t} \leq 300 \right)
\]

Allowable beam and torsional shear stresses, \( \sigma_v \):
\[
\sigma_v = f_a \ 0.4\sigma_y
\]

where,
\[
f_a = \text{allowable stress factor, } \approx 1.0 \text{ for load cases defined in Ch 4, 3}
\]
\[
L_p = \text{un-braced length of cylindrical member, to be taken as the modelled distance between nodal joints at each end of the member, m}
\]
\[ E = \] Young’s Modulus of elasticity, N/mm\(^2\)
\[ t = \] wall thickness of cylindrical member, m
\[ D = \] outside diameter of cylindrical member, m
\[ K = \] effective length factor of cylindrical member, to be taken as 0.8
\[ r = \] radius of gyration of cross section of cylindrical member, m
\[ \sigma_f = \] minimum of the yield stress \( \sigma_y \) and the local buckling stresses \( \sigma_{xc} \), N/mm\(^2\).
\[ \sigma_{xe} = \] elastic local buckling stress, N/mm\(^2\): \( \sigma_{xe} = 0.6E/tD \)
\[ \sigma_{xe} = \] inelastic local buckling stress, N/mm\(^2\): \[
\begin{align*}
\sigma_{xc} = \sigma_y & \quad \text{for } (D/t)^2 \leq 60 \\
\sigma_{xc} = \left[1.64 - 0.23(D/t)^2\right] & \quad \text{for } (D/t)^2 > 60
\end{align*}
\]
\[ C_c = \] compression factor
\[ C_c = \sqrt{\frac{2\pi^2E}{\sigma_f}} \]
\[ \sigma_p = \] yield stress, N/mm\(^2\), to be taken as 0.2% of the Proof strength in the case of stainless steel. The temperature dependent value of yield stress may be used.

### 1.2 Stress assessment for cylindrical members

For all positions along the length of each main tubular column and cross brace, the combined axial and bending utilisation factor is to satisfy the following:

\[ F_{ax} + F_b \leq 1.0 \]

where:

- \( F_{ax} = \) axial utilisation factor defined as:
  - at the ends \( F_{ax} = \left| \frac{\sigma_{ax}}{\sigma_t} \right| \)
  - for all other locations i.e., 0.1\(L_p\), 0.25\(L_p\), 0.5\(L_p\), 0.75\(L_p\) and 0.9\(L_p\)
    \[ F_{ax} = \frac{\sigma_{ax}}{\sigma_t} \quad \text{when } \sigma_{ax} \geq 0.0 \]
    \[ F_{ax} = \frac{\sigma_{ax}}{\sigma_{xe}} \quad \text{when } \sigma_{ax} < 0.0 \]

- \( F_b = \) bending utilisation factor defined as:
  - at the ends \( F_b = \sqrt{\frac{\sigma_{pb}^2 + \sigma_{ph}^2}{\sigma_b^2}} \)
  - for all other locations i.e., 0.1\(L_p\), 0.25\(L_p\), 0.5\(L_p\), 0.75\(L_p\) and 0.9\(L_p\)
    \[ F_b = \sqrt{\frac{\sigma_{pb}^2 + \sigma_{ph}^2}{\sigma_b^2}} \quad \text{when } \sigma_{ax} \geq 0.0 \text{ or } \left( \sigma_{ax} < 0.0 \text{ and } F_{ax} \leq 0.15 \right) \]
    \[ F_b = \sqrt{\left[ \frac{C_{mp}\sigma_{pb}}{1-\sigma_{ax}/\sigma_t} \right]^2 + \left[ \frac{C_{mp}\sigma_{ph}}{1-\sigma_{ax}/\sigma_t} \right]^2} \quad \text{when } \sigma_{ax} < 0.0 \text{ and } F_{ax} > 0.15 \]
where,

- $\sigma_{ax}$ = axial stress of member from FE analysis
- $\sigma_{ipb}$ = in-plane bending stress of member from FE analysis
- $\sigma_{opb}$ = out-of-plane bending stress of member from FE analysis
- $C_{mip}, C_{mop}$ = reduction factors, to be taken as 0.85
- $\sigma_a, \sigma_c, \sigma_b$ = allowable stresses as given in 3.1

$$\sigma_v = \frac{12\pi^2 E}{23(KL_p / r)^2}$$

For all positions along the length of each main tubular column and cross brace, the shear stress and torsional shear stress are to satisfy the following:

$$\frac{\sigma_{vb}}{\sigma_v} \leq 1.0 \text{ and } \frac{\sigma_{vt}}{\sigma_v} \leq 1.0$$

where,

- $\sigma_{vb}$ = beam shear stress is defined as: $$\sigma_{vb} = \frac{F_t}{0.5A}$$
- $\sigma_{vt}$ = torsional shear stress is defined as: $$\sigma_{vt} = \frac{M_t(D/2)}{I_p}$$

- $F_t$ = transverse shear force from FE analysis, MN
- $M_t$ = torsional moment from FE analysis, MNm
- $I_p$ = polar moment of inertia, m$^4$
- $A$ = cross section area, m$^2$

### 1.3 Hoop buckling stress check

For cross braces which are fluid tight and are completely or partial immersed in the LNG fluid, a hoop buckling stress check is to be carried out. This is to assess the local buckling strength of the cross brace tubes when subject to hydrostatic pressure from the LNG fluid.

#### 1.3.1 Hoop buckling due to hydrostatic pressure only

The hoop stress $\sigma_h$ due to hydrostatic pressure is to satisfy the following:

$$\frac{\sigma_h}{(\sigma_{hc} / f_h)} \leq 1.0$$

where:

- $f_h$ = safety factor for hoop compression, to be taken as 2.0
- $\sigma_h$ = hoop compressive stress (N/mm$^2$), $\sigma_h = PD / 2t$
- $P$ = hydrostatic pressure (N/mm$^2$), $P = 10^{-6} \rho g H$
- $\rho$ = density of LNG fluid (kg/m$^3$)
- $g = 9.81$ (m/s$^2$)

---

1 In-plane bending induces stresses at the locations T and B shown in Ch 5, Fig 5.2.
2 Out-of-plane bending induces stresses at the locations F and A shown in Ch 5, Fig 5.2.
\[ H = \text{distance from the mean fluid level to the point on the cross brace being assessed (m)} \]

\[ \sigma_{hc} = \text{Critical hoop buckling stress} \quad (\text{N/mm}^2) \]

**Elastic buckling**

\[ \sigma_{hc} = \sigma_{he} \quad \text{for } \sigma_{he} \leq 0.55\sigma_y \]

**Inelastic buckling**

\[ \sigma_{hc} = 0.45\sigma_y + 0.18\sigma_{he} \quad \text{for } 0.55\sigma_y < \sigma_{he} \leq 1.6\sigma_y \]

\[ \sigma_{hc} = \frac{1.31\sigma_y}{1.15 + (\sigma_y / \sigma_{he})} \quad \text{for } 1.6\sigma_y < \sigma_{he} \leq 6.2\sigma_y \]

\[ \sigma_{hc} = \sigma_y \quad \text{for } \sigma_{he} > 6.2\sigma_y \]

\[ \sigma_{he} = \text{elastic hoop buckling stress} \quad (\text{N/mm}^2), \quad \sigma_{he} = 2C_hEt / D \]

\[ C_h = \text{critical hoop buckling coefficient, where:} \]

\[ C_h = 0.44(t / D) \quad \text{for } M \geq 1.6(D / t) \]

\[ C_h = 0.44(t / D) + \frac{0.21(D / t)^3}{M^4} \quad \text{for } 0.825(D / t) \leq M < 1.6(D / t) \]

\[ C_h = 0.736/(M - 0.636) \quad \text{for } 3.5 \leq M < 0.825(D / t) \]

\[ C_h = 0.755/(M - 0.559) \quad \text{for } 1.5 \leq M < 3.5 \]

\[ C_h = 0.8 \quad \text{for } M < 1.5 \]

\[ M = \text{geometric factor} \quad M = \frac{L_b}{D} (2D / t)^{1/2} \]

\[ L_b = \text{braced length which may be taken as } L_p \quad (\text{m}), \text{ see 3.1} \]

\[ D, t \text{ are given in 3.1} \]

**1.3.2 Buckling stress due to combined dynamic, temperature and hydrostatic loads**

**1.3.2.1 Cross braces in tension**

\[ a) \quad \text{For cross braces with induced tensile axial stress due to the applied loads (but see also 1.3.2.2(c)), the following criterion is to be satisfied when combined with hoop stress:} \]

\[ S_A^2 + S_H^2 + 2\nu S_A S_H \leq 1.0 \]

**where:**

\[ S_A = \frac{\left| \sigma_{amax} \right| + \left| \sigma_{bmax} \right| - \left| 0.5\sigma_{hc} \right|}{\sigma_y} f_t \]

\[ S_H = \frac{\sigma_{he}}{\sigma_{hc}} f_h \]

\[ \nu = \text{Poisson’s ratio} \]

\[ \sigma_{amax} = \text{maximum tensile axial stress along the length of the cross brace (N/mm}^2\) \]

\[ \sigma_{bmax} = \text{maximum bending stress along the length of the cross brace (N/mm}^2\), \text{i.e.} \]

\[ = \sqrt{\sigma_{hp}^2 + \sigma_{obp}^2} \]
\[ f_t \quad = \quad \text{safety factor for axial tension, to be taken as 1.67} \]

\[ \sigma_n, \sigma_h \quad \text{are as given in 1.3.1} \]

b) if \( |\sigma_{b\text{max}}| > |\sigma_{a\text{max}}| + 0.5|\sigma_h| \), then paragraph 1.3.2.2 a) is also to be satisfied, even though the cross brace is in tension. The absolute value of the axial stress in the cross brace member is to be used to determine \( \sigma_{a\text{max}} \).

### 1.3.2.2 Cross braces in compression

a) For the cross braces with induced **compressive** axial stress due to the applied loads (but see also 1.3.2.1(b)), the following criterion is to be satisfied when combined with hoop stress:

\[
\frac{\left|\sigma_{a\text{max}}\right| + 0.5|\sigma_h|}{\sigma_{sc}} \cdot f_c + \frac{|\sigma_{b\text{max}}|}{\sigma_y} \cdot f_b \leq 1.0
\]

where:

\( \sigma_{a\text{max}} \) = maximum axial compressive stress along the length of the cross brace (N/mm²)

\( f_c \) = safety factor for axial compression, to be taken as 1.67

\( f_b \) = safety factor for bending: \( f_b = \frac{\sigma_y}{\sigma_b} \)

\( \sigma_{b\text{max}} \) is as given in 1.3.2.1

\( \sigma_n, \sigma_h \) are as given in 1.3.1

\( \sigma_b \) is as given in 1.1

b) Additionally, if \( \sigma_x > 0.5\sigma_{ha} \), the following criterion is also to be satisfied:

\[
\left( \frac{\sigma_x - 0.5\sigma_{ha}}{\sigma_{au} - 0.5\sigma_{ha}} \right)^2 + \left( \frac{\sigma_h}{\sigma_{ha}} \right)^2 \leq 1.0
\]

where:

\( \sigma_{au} = \frac{\sigma_{xu}}{f_c} \) and \( \sigma_{ha} = \frac{\sigma_{hu}}{f_h} \)

\( \sigma_x \) = \( \left|\sigma_{a\text{max}}\right| + \left|\sigma_{b\text{max}}\right| + 0.5|\sigma_h| \)

\( \sigma_{xu} \) is as given in 1.2

\( \sigma_{hu} \) is as given in 1.3.1

c) If \( |\sigma_{b\text{max}}| > |\sigma_{a\text{max}}| + 0.5|\sigma_h| \), then the requirement of paragraph 1.3.2.1 a) is to be satisfied even though the cross brace is in compression. The absolute value of axial stress in the cross brace member is to be used to determine \( \sigma_{a\text{max}} \).
Section 2: Pump Tower Joints - Strength Assessment

Assessment of joint strength is based on the API code. This code states that the analysis of a joint may be based on the assessment of either: a) nominal loads in the brace, or b) punching shear capability. It is stated that the results derived from these two approaches are equivalent. For this procedure, assessment is to be based on the most conservative of the two results obtained using these approaches. Both assessment methods are re-produced in 2.1 and 2.2.

Figure 5.1 shows a simple tubular joint. Such joints are classified as K, T, Y or X type according to the structural configuration. The capacity depends on joint type, geometric parameters and the axial and bending stresses in the tubular members. The punching strength of each connection between the Pump Tower tubular column and the cross brace and connections between the cross braces are to be assessed as given in 2.1 and 2.2.

![Figure 5.1: Details of a simple tubular joint](image)

2.1 Nominal loads assessment method

The forces and moments in the braces at the joints are to satisfy the criteria given below:

Moment check:

\[
\left( \frac{M}{M_{,ipb}} \right)^2 + \left( \frac{M}{M_{,opb}} \right)^2 \leq 1.0
\]

Combined check:

\[
\left( \frac{P}{P_a} \right) + \frac{2}{\pi} \arcsin \left( \sqrt{ \left( \frac{M}{M_{,ipb}} \right)^2 + \left( \frac{M}{M_{,opb}} \right)^2 } \right) \leq 1.0
\]

where:

\( P \) = axial force in the cross brace in way of the joint from FE analysis (N),

\( \sigma_{ax} \) = axial stress in the cross brace at the joint from FE analysis (N/mm²)

\( M \) = in-plane or out-of-plane bending moment in the cross brace in way of the joint (Nmm),

\( \sigma_{ipb} \) or \( \sigma_{opb} \), as appropriate

\( Z \) = the section modulus of the cross brace (mm³)

\( \sigma_{ipb}, \sigma_{opb} \) = in-plane bending (ipb), and out-of-plane bending (opb) stresses in the cross brace at the joint from FE analysis (N/mm²)

\( P_a \) = allowable axial load on cross brace, (KN)

\( \theta \) = Brace angle (measured from chord), deg

\( g \) = Gap, mm

\( t_c \) = Cross brace thickness, mm

\( t_p \) = Pump Tower tubular column thickness, mm

\( D_c \) = Cross brace diameter, mm

\( D_p \) = Pump Tower tubular column diameter, mm

\( \beta = \frac{D_c}{D_p}, \quad \gamma = \frac{D_p}{2t_p}, \quad \alpha = \frac{t_c}{t_p} \)
\[ P_a = f_a Q_f Q_f \frac{\sigma_{yc} t_p}{1.7 \sin \theta} \]

\[ M_a = f_a Q_f Q_f \frac{\sigma_{yc} t_p^2}{1.7 \sin \theta} \left(0.8 D_c\right) \]

\[ f_a = \text{allowable stress factor} = 1.0 \]

\[ \sigma_{yc} = \text{the yield stress of the chord member at the joint (or 2/3 of the ultimate tensile strength if less) (N/mm}^2) \]

\[ t_p, D_c, \theta, \beta \text{ and } \gamma \text{ are defined in Figure 5.1} \]

\[ Q_f = \text{the ultimate strength factor which varies with the joint and load type, (see Table 5.1).} \]

\[ Q_f = 1.0 - \min(1, 0.5 \gamma R^2) \]

\[ \lambda = 0.030 \text{ for cross brace axial stress} \]
\[ = 0.045 \text{ for cross brace in-plane bending stress} \]
\[ = 0.021 \text{ for cross brace out-of-plane bending stress} \]

\[ R = f_a \frac{\sqrt{\sigma_{ax}^2 + \sigma_{ipb}^2 + \sigma_{opb}^2}}{0.6 \sigma_{yc}} \]

\[ \sigma_{ax}, \sigma_{ipb}, \text{and } \sigma_{opb} \text{ are the axial, in-plane bending (ipb), and out-of-plane bending (opb) stresses in the pump tower column in way of the joint.} \]

### 2.2 Punching shear assessment method

The following criteria are to be satisfied for combined axial and bending stresses in the brace at the joint:

**Moment check:**

\[ \left( \frac{\sigma_p}{\sigma_{pa}} \right)_{ipb}^2 + \left( \frac{\sigma_p}{\sigma_{pa}} \right)_{opb}^2 \leq 1.0 \]

**Combined check:**

\[ \left| \frac{\sigma_p}{\sigma_{pa}} \right|_{ax} \leq 2 \pi \arcsin \left( \sqrt{\left( \frac{\sigma_p}{\sigma_{pa}} \right)_{ipb}^2 + \left( \frac{\sigma_p}{\sigma_{pa}} \right)_{opb}^2} \right) \leq 1.0 \]

The punching shear at a joint is calculated separately for each load component in the brace from:

\[ \sigma_p = a \sigma \sin \theta \]

where,

\[ \sigma \text{ is either the axial, in-plane bending or out-of-plane bending stress in the cross brace in way of the joint, to be taken from the FE analysis.} \]

Allowable punching shear stress in the wall of the Pump Tower column is defined as:

\[ \sigma_{pa} = f_a Q_f Q_f \frac{\sigma_{yc}}{0.6 \gamma} \]

but is not to be taken greater than \(0.4 \sigma_y\)
where:

\( f_a, \sigma_{yc}, Q_h, \gamma \) are as defined in 2.1

\( Q_q \) is as given in Table 5.1

**Table 5.1: Values for Strength Factors \( Q_q \) and \( Q_u \)**

<table>
<thead>
<tr>
<th>Joint Type</th>
<th>( Q_q )</th>
<th>( Q_u )</th>
<th>( Q_q )</th>
<th>( Q_u )</th>
<th>( Q_q )</th>
<th>( Q_u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strength Factors</td>
<td>( Q_q )</td>
<td>( Q_u )</td>
<td>( Q_q )</td>
<td>( Q_u )</td>
<td>( Q_q )</td>
<td>( Q_u )</td>
</tr>
<tr>
<td>Loads in braces</td>
<td>(1.10+0.20/( \beta ))( Q_g )</td>
<td>(3.4+19( \beta ))( Q_g )</td>
<td>(1.10+0.20/( \beta ))</td>
<td>(3.4+19( \beta ))</td>
<td>(1.10+0.20/( \beta ))</td>
<td>(3.4+19( \beta ))</td>
</tr>
<tr>
<td>Axial tension</td>
<td>(3.72+0.67/( \beta ))</td>
<td>(3.4+19( \beta ))</td>
<td>(3.72+0.67/( \beta ))</td>
<td>(3.4+19( \beta ))</td>
<td>(3.72+0.67/( \beta ))</td>
<td>(3.4+19( \beta ))</td>
</tr>
<tr>
<td>In-plane bending</td>
<td>(1.37+0.67/( \beta ))( Q_b )</td>
<td>(3.4+7( \beta ))( Q_b )</td>
<td>(1.37+0.67/( \beta ))( Q_b )</td>
<td>(3.4+7( \beta ))( Q_b )</td>
<td>(1.37+0.67/( \beta ))( Q_b )</td>
<td>(3.4+7( \beta ))( Q_b )</td>
</tr>
</tbody>
</table>

\( Q_{\beta} = \frac{0.3}{\beta(1-0.833\beta)} \) for \( \beta > 0.6 \)

\( Q_{\beta} = 1.0 \) for \( \beta \leq 0.6 \)

\( \beta \) is given in Figure 5.1

\( Q_g = 1.8 - \frac{0.1g}{t_p} \) for \( \gamma \leq 20 \)

\( Q_g = 1.8 - \frac{4g}{D_p} \) for \( \gamma > 20 \)

\( Q_g \) is not to be less than 1.0

2.3 Welds at connections

Welds at the ends of tubular members are to be in accordance with 3.9.1 and are to have a capacity greater than or equal to the lesser of:

a) the strength of the cross brace member based on yield and
b) the strength of the chord member based on punching shear.

Fillet welds are to be sized such that the shear stress in the weld throat resulting from axial and bending stresses, does not exceed 0.35\( \sigma_y \). See also 3.9

■ Section 3: Pump Tower Joints - Fatigue Check

Fatigue analysis of the joints is to be carried out according to the requirements of this section.

Fatigue calculations are normally to be carried out for stresses calculated using FE analysis (see Ch 4).

The fatigue analysis is to be based on the assumption that the ship spends half of its operational life in ballast and half fully loaded. The total fatigue damage is obtained by adding the fatigue damage from each load case.

The contributing load cases and the proportions of the each load case considered may need to be amended if more onerous conditions are dictated by the operational profile of the ship e.g. if the ship is to be routinely operated with the tanks partially filled during loaded voyages.
3.1 Fatigue Damage Calculation

The fatigue damage factor $D$ is to be calculated using linear damage summation rule. This may be expressed in integral form as follows:

$$D = N' \int \left( \frac{p(S)}{N(S)} \right) dS$$

where:
- $N'$ is the required design life, translated into number of cycles, for the ship (see 3.2.1)
- $N(S)$ is the design S-N curve (see 3.6)
- $p(S)$ is the long-term probability density function (see 3.5).

3.2 Design Parameters & Acceptance Criteria

3.2.1 Minimum Requirements

A minimum fatigue life, $N$, of 20 years is required in association with a damage factor, $D$, of 0.5 and utilisation factor, $U_f$, of 0.82. This criterion corresponds to North Atlantic service.

The utilisation factor $U_f$ is calculated as follows based on the assumption that a ship spends 150 days at sea per year in each of the loading cases F1 and F2 described in 3.3:

$$U_f = \frac{2 \times 150}{365} = 0.82$$

The number of stress cycles in the design life $N'$ may be calculated from:

$$N' = N' \times 31536000 \times U_f / T$$

where $T$ is the zero up-crossing period, which may be taken as the ship natural period of Roll.

3.2.2 Owners Requirements

A more onerous criterion may be necessary to meet the requirements of individual design specifications. In this case, the shape parameter, $k$, in 3.5, may be determined based on acceptable design studies and the utilisation factor, $U_f$, in 3.2.1, may be re-assessed to reflect specific fatigue life or loading condition requirements. The design life, $N$, is to be as given in the design specification if that is greater than 20 years.

3.3 FE Analysis.

The nominal stresses, $\sigma_{ax}$, $\sigma_{ipb}$ and $\sigma_{opb}$, required as input to the calculation of the joint hot spot stresses (see 3.4), are to be derived using the finite element model described in Chapter 4. The load cases to be considered are as follows:

**Load case $F_{1n}$ – Normal Ballast Condition**

Ship natural roll period case with cargo at fill height $F = 5\%$ H, comprising the following load components:

(a) Hydrodynamic Fluid forces for a fill height 5% H, see Ch 3, 3
(b) Transverse inertial loading for a fill height 5% H, see Ch 4, 2

If it is intended to regularly trade in a normal ballast condition with a fill level of more than 5% H in any tank, it will be necessary to adjust this condition to reflect the anticipated fill level. In no case is a fill height of less than 5% H to be used.

**Load case $F_{2n}$ – Normal Full Load condition**

Ship natural roll period case with cargo at fill height $F = 95\%$ H, comprising the following load components:

(a) Hydrodynamic Fluid forces for a fill height 95% H, see Ch 3, 3
(b) Transverse inertial loading, for a fill height of 95% H, see Ch 4, 2
The loading conditions used for the fatigue assessment are to represent conditions that will be most commonly used throughout the ship’s life. If it is expected that the ship will trade for a significant portion of its life in partially loaded conditions, then this will need to be special considered.

The load components for cases $F_{1n}$ and $F_{2n}$ are to be combined as shown in the following table:

<table>
<thead>
<tr>
<th>Load component</th>
<th>Load cases $F_{11}$ &amp; $F_{21}$</th>
<th>Load cases $F_{12}$ &amp; $F_{22}$</th>
<th>Load cases $F_{13}$ &amp; $F_{23}$</th>
<th>Load cases $F_{14}$ &amp; $F_{24}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) hydrodynamic LNG forces</td>
<td>+ve fluid forces</td>
<td>+ve fluid forces</td>
<td>-ve fluid forces</td>
<td>-ve fluid forces</td>
</tr>
<tr>
<td>b) inertial forces</td>
<td>+ve inertial force</td>
<td>-ve inertial force</td>
<td>-ve inertial force</td>
<td>+ve inertial force</td>
</tr>
</tbody>
</table>

Notes:
- Load components a) and b) are as described in preceding paragraphs
- +ve and –ve load components are defined below:
  - +ve fluid forces are fluid forces in the direction of +ve velocity, as calculated using Ch3, 3
  - -ve fluid forces are fluid forces in the direction of -ve velocity, as calculated using Ch3, 3
  - +ve inertial forces are consistent with roll to starboard
  - -ve inertial forces are consistent with roll to port

3.4 Stress Ranges

The fatigue analysis is to be based on the corrected hot spot stress range, $S_R$, at the weld toe, taking into account both the axial and bending stress components. The corrected hot spot stress range is to be obtained as the product of corresponding hot spot stress ranges, joint stress concentration factors (SCF) and correction factors for thickness and Young’s modulus as follows:

$$S_R = K_T K_m S_{HR}$$

where:
- $K_T$ = correction factor for Thickness effect, see 3.7
- $K_m$ = correction factor for Young’s Modulus, see 3.8
- $S_{HR}$ = hot spot stress range. This is to be calculated as a combination of nominal axial and bending stress components $\sigma_{ax}$, $\sigma_{bb}$, $\sigma_{opb}$ derived from FE analysis and applying a stress concentration factor, $K_{joint}$
  - $S_{HR}$ is to be taken as the maximum value calculated from any of the load cases $F_{11}$, $F_{12}$, $F_{13}$ or $F_{14}$ for the normal ballast load condition.
  - Similarly, $S_{HR}$ is to be taken as the maximum value calculated from any of the load cases $F_{21}$, $F_{22}$, $F_{23}$ or $F_{24}$ for the normal loaded load condition.
- $K_{joint}$ = SCF associated with the joint geometry

For fatigue assessment of the weld toe, the appropriate value of $K_{joint}$, the SCF associated with the joint geometry, is to be derived in accordance with the formulations given in Appendix I and the T’ Class (tubular) S-N curve defined in 3.6 is to be used.

For fatigue assessment of weld root, $K_{joint}$ is to be taken as 1.0 and the W Class S-N curve defined in 3.6 is to be used. Additional requirements for the weld root are given in 3.9.1.

The fatigue assessment is to be carried out at a minimum of four locations around the cross-brace to PT main column joints as shown in Figure 5.2. The assessment is also to be applied to any other
locations which are considered to be critical. The hot spot stress ranges, $S_{hr}$, are to be taken as follows:

- **T**: Top location
  
  $$S_{hR} = 2K_{\text{joint}ax} \sigma_{ax} + K_{\text{joint}ip} \sigma_{ipb}$$

- **B**: Bottom location
  
  $$S_{hR} = 2K_{\text{joint}ax} \sigma_{ax} - K_{\text{joint}ip} \sigma_{ipb}$$

- **F**: Forward location
  
  $$S_{hR} = 2K_{\text{joint}ax} \sigma_{ax} + K_{\text{joint}op} \sigma_{opb}$$

- **A**: Aft location
  
  $$S_{hR} = 2K_{\text{joint}ax} \sigma_{ax} - K_{\text{joint}op} \sigma_{opb}$$

where:

- $\sigma_{ax}, \sigma_{ipb}, \sigma_{opb}$ are stresses obtained from FE analysis for the load cases $F_1$ and $F_2$ described in 3.3 and,

- $K_{\text{joint}ax}, K_{\text{joint}ip}$, and $K_{\text{joint}op}$ are the corresponding stress concentration factors.

![Diagram of fatigue locations](Figure 5.2 Fatigue locations to be considered)

### 3.5 Stress History

The stress history to be applied takes the form of a two-parameter Weibull probability distribution, as follows:

Probability density function of the stress range $p(S)$:

$$p(S) = \frac{k}{a_s} \left( \frac{S}{a_s} \right)^{k-1} \exp \left( - \left( \frac{S}{a_s} \right)^k \right)$$

where:

- $k$ is the shape parameter of the Weibull distribution, to be taken as 1.0.
- $a_s = \frac{S_R}{(\ln N_s)^{\frac{1}{k}}}$ is the scale parameter of the Weibull distribution, assuming a probability of occurrence of $\frac{1}{N_s} = 10^{-i}$
- $S_R$ is the maximum hot spot stress range, as defined in 3.4.
3.6 Design S-N curve

The basic Design S-N curve to be used is the T' Class (tubular), in air, as follows:

\[ \log_{10}(N) = \log_{10}(K_2) - m \times \log_{10}(S) \]

This basic Design S-N curve, for which the details are given in Table 3.2, is applicable for:

(i) members with thickness less than the reference thickness, \( t_{\text{ref}} \) given in the Table

(ii) normal ferritic steel material at ambient temperature with a Young's modulus of elasticity \( (E) \) equal to \( 2.06 \times 10^5 \text{ N/mm}^2 \).

Hence for thicker sections or for stainless steel material, correction factors are normally applied to adjust the design S-N curve. However, in this procedure, these factors are already included in the stress range equation as described in 3.4 and hence no further modification of the S-N curve is required (see 3.4, 3.7 and 3.8).

<table>
<thead>
<tr>
<th>Class</th>
<th>Environment</th>
<th>( \log_{10}(K_2) )</th>
<th>m</th>
<th>( S_s, N/\text{mm}^2 )</th>
<th>( N_0, \text{Cycles} )</th>
<th>( t_{\text{ref}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T'</td>
<td>Air/LNG</td>
<td>12.476, 16.127</td>
<td>3</td>
<td>67</td>
<td>( 10^7 )</td>
<td>16</td>
</tr>
<tr>
<td>W</td>
<td>Air/LNG</td>
<td>11.204, 14.007</td>
<td>3</td>
<td>25.2</td>
<td>( 10^7 )</td>
<td>22</td>
</tr>
</tbody>
</table>

where:

- \( N_j \) is the no. of cycles at which slope of the S-N curve changes from \( m=3 \) to \( m=5 \),
- \( S_s \) is the corresponding reference stress range
- \( K_2 \) reflects two standard deviations below the mean line
- \( t_{\text{ref}} \) is the reference thickness of the S-N curve

3.7 Correction Factor for Thickness

For members with thickness greater than \( t_{\text{ref}} \), it is necessary to either adjust the basic design S-N Curve, or to modify the stress range used in the fatigue calculation. In this procedure the second option is adopted (see 3.4). The correction factor \( K_T \) is defined as follows:

\[ K_T = \begin{cases} 1 & \text{for } t_1 < t_{\text{ref}} \\ \left( \frac{t_1}{t_{\text{ref}}} \right)^{0.3} & \text{for } t_1 > t_{\text{ref}} \end{cases} \]

where:

- \( t_1 \) = the thickness of the thinner member, in mm
- \( t_{\text{ref}} \) = the reference thickness of the S-N curve (see Table 3.2)

3.8 Correction Factor for Young’s Modulus (E)

For material other than that defined in 3.6, it is necessary to either adjust the basic design S-N Curve, or to modify the stress range used in the fatigue calculation. In this procedure the second option is adopted (see 3.4). The correction factor \( K_m \) is defined as follows:
\[ K_m = \frac{2.06 \times 10^5}{E} \]

where:
\[ E = \text{Young's modulus of elasticity for the material under consideration, in } N/mm^2 \]

Hence for stainless steels AISI 316L and 304L having Young's modulus, \( E = 1.93 \times 10^5 \frac{N}{mm^2} \),

\[ K_m = \frac{2.06 \times 10^5}{1.93 \times 10^5} = 1.067 \]

3.9 Other considerations

3.9.1 Weld type

Full penetration welds

The use of full penetration welds is recommended for all tubular brace to chord connections.

For full penetration welded joints, the most critical location for fatigue damage is the weld toe.

If a full penetration weld fails to satisfy the agreed acceptance criterion, the pipe wall thickness is to be increased to reduce the nominal stress to an acceptable level. Account is to be taken of the thickness correction factor in this calculation.

Partial penetration and fillet welds

In load carrying partial penetration or fillet welded joints, where cracking could occur in the weld throat, fatigue calculations are to be performed to demonstrate that the allowable damage ratio is not exceeded at the weld root as well as the weld toe.

In addition, the potential for root cracking is to be avoided by the use of appropriate weld processes and procedures, e.g. use of TIG welding for the root run.

When checking the weld root, the relevant stress range is to be taken as the maximum range of shear stress induced in the weld material. In calculating this stress range, \( K_{\text{joint}} \) is to be taken as 1.0, \( K_i \) is as given in 3.7 using the relevant \( t_{\text{ref}} \) and \( K_m \) is as given in 3.8.

This shear stress range is to be used in combination with the W-curve parameters specified in Table 3.2.

If any of the welds fail to meet the acceptance criterion, the weld throat is to be increased and the calculation repeated.

3.9.2 Effect of Undercut

Effect of undercut is already included to a limited extent in the S-N curves. According to "IIW document XIII-1965-03 / XV-1127-03, Recommendations for Fatigue Design of Welded Joints and Components, July 2004", the ratio of maximum allowable undercut to plate thickness, \( u/t \), for fillet welds is 0.05. If \( u/t \) is greater than 0.05 or the depth of the undercut is greater than 1mm, the undercut is to be considered as a crack-like defect which should be removed using a procedure approved by Lloyd's Register.
Section 4: Pump Tower Base Support – Allowable Deflections

The reinforcement of the inner bottom immediately in way of the Pump Tower Base Support (PTBS) is to be sufficiently rigid to restrict lateral displacement of the PTBS to a low value at the primary and secondary membrane connections. This is in order to protect the membranes against higher deflections than can be tolerated by the containment system design. The allowable deflections are to be obtained from the containment system designers. Compliance with these allowable values is to be confirmed for all load cases.

Section 5: Pump Tower Base Support - Allowable Stresses

For all load cases, the maximum allowable membrane (ie plate element mid thickness) stresses in the Pump Tower Base Support and guide assembly and in the Pump Tower Base Support Structure are:

- $0.85\sigma_y$ for Von Mises stress
- $0.75\sigma_y$ for direct stress
- $0.35\sigma_y$ for shear stress

where:

- $\sigma_y$ = yield stress, N/mm$^2$, to be taken as 0.2% of the Proof stress in the case of stainless steel.
- Temperature dependent values of yield stress may be used.

For any synthetic material used in the contact surfaces, the manufacturer’s allowable stresses and other limiting design criteria are to be followed. Here the acceptance criteria are to take account of the creep.

Section 6: Pump Tower Base Support - Fatigue Check

6.1 FE Modelling

The results of the Pump Tower Base Support strength analysis are to be reviewed to identify which part(s) of the structure are susceptible to fatigue failure. The model is then to be refined to incorporate a mesh size of $t \times t$, where $t$ represents the plate thickness in way of location(s) found to be susceptible.

6.2 Fatigue Damage Calculations

This Sub-Section applies to Pump Tower Base Supports which are fabricated from plating and welded to the inner bottom. If the construction incorporates a strongly built wooden structure bolted to the inner bottom, such as in the earlier TGZ Mk III system, the methods by which fatigue strength is to be evaluated will be specially considered.

The fatigue calculation required is similar to that described in Section 5, using FE analysis results for the Pump Tower Base Support at the location being considered. The differences in the fatigue assessment approach and requirements from that described in Section 5 are given below:

a) The basic design S-N curve to be used for the assessment of fatigue damage at the weld toe is the D class in air. Parameters are given in Table 5.3.

<table>
<thead>
<tr>
<th>Class</th>
<th>Environment</th>
<th>$\log_{10}(K_i)$</th>
<th>m</th>
<th>$S_o$, N/mm$^2$</th>
<th>$N_o$, cycles</th>
<th>$t_{ref}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>Air/LNG</td>
<td>12.182</td>
<td>3</td>
<td>53.4</td>
<td>$10^7$</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>15.637</td>
<td>3</td>
<td>5</td>
<td>25.2</td>
<td>$10^7$</td>
<td>22</td>
</tr>
</tbody>
</table>

See Table 3.2 for details of symbols
b) The hot spot stress range, \( S_R \), is given by:

\[
S_R = 2 \, K_T \, K_m \, \sigma_{fem}
\]

Where,

- \( K_T \) = correction factor for thickness as given in 3.7
- \( K_m \) = correction factor for Young’s Modulus as given in 3.8
- \( \sigma_{fem} \) = principal stress within an arc described by lines ±45º to the line perpendicular to the anticipated crack. This stress is to be extracted from the t x t element adjacent to the location being considered.

If the Pump Tower Base Support is not manufactured from stainless steel, the values of \( K_T \) and \( K_m \) are to be specially considered. The treatment of any undercut is to be as described in 3.9.2.

c) Weld type

**Full penetration welds**

The use of full penetration welds is recommended for all connections in fatigue critical locations. For full penetration welded joints made from both sides, it is only necessary to assess the fatigue damage at the weld toe.

**Partial penetration welds, welds made from one side and fillet welds,**

The use of partial penetration welds, welds made from one side and fillet welds, is to be avoided in areas which are considered fatigue critical.

However, if such welds are used, the potential for root cracking is to be eliminated and fatigue damage calculations are to be performed for both the toe and root locations to establish this.

For fatigue calculation at the weld root, the relevant stress range is the maximum range of shear stress induced in the weld throat.

This shear stress range is to be used in combination with the W-curve specified in Table 3.2. If any of the welds fail to comply with the acceptance criterion, the weld throat is to be increased and the calculation repeated.
Chapter 6: Vibration Check

Section 1. General
Section 2. Modelling
Section 3. Assessment
Section 4. Re-analysis

Section 1: General

Normal mode analysis is to be carried out for the finite element model of the Pump Tower in two conditions:

- Dry, when the ship is in the ballast condition. This condition will give the upper limit of the natural frequency range of the structure.
- Wet, when the tank is filled to its maximum limit (95% H). This condition will produce the lower limit of the natural frequency range of the structure.

For other filling levels between 0 and 95%H, the natural frequency will be bounded by the above values.

Section 2: Modelling

In principle, only the model of the Pump Tower itself, see Chapter 4, 1.2 is required to be used.

The boundary conditions to be used with this model are:

- **In way of guide assembly between the Pump Tower and PTBS:**
  Constrained in rotation about the vertical axis of the tower and in translation in the longitudinal and transverse directions. All other rotations and translations are free of constraint.

- **In way of the Pump Tower Top Support (PTTS):**
  The PTTS arrangements depend on the type of containment system fitted in the vessel. For containment systems without a large opening in way of the Pump Tower and in which the PTTS connects to the inner trunk deck, such as the GTT No 96 type, the upper connections of the main vertical tubular columns to the inner and outer trunk deck may be represented by boundary conditions as follows:
  - Vertical, transverse and longitudinal translation constraints at the inner deck level.
  - All other translations and rotations at the inner trunk deck and outer trunk deck level are free of constraint.

  For containment systems having a large liquid dome opening in the trunk deck in way of the Pump Tower location, such as Technigaz MK III design, then:
  - Plate and line elements are to be used to represent the hatch cover design at outer trunk deck level. It is not necessary to include the dome opening coaming.
  - The hatch cover model is to be constrained in all translations and rotations at its boundaries.
The Pump Tower vertical members are to be connected to the hatch cover stiffening in all degrees of freedom.

In the wet condition, the influence of the fluid external to the pipe structure is to be taken into account in the form of added extra masses to the pipes. These masses are to be calculated from the standard formulation for cylinders shown below and are applied as non-structural masses to the bar elements in the model. Fluid in the Pump Tower pipes is also to be included as non-structural masses.

Fluid added mass per unit length for cylinders is given by:

\[ m = \rho \pi \frac{d^2}{4} \]

where:
- \( \rho \) = fluid density
- \( d \) = diameter of cylinder

### Section 3: Assessment

For steam turbine propulsion, engine vibration is not considered as a possible exciting frequency but, for other forms of propulsion (e.g. direct drive diesel), engine excitation frequencies are to be considered.

For propellers, only blade frequency needs to be considered. Propeller excitation frequency is to be based on a range of RPM, i.e. the upper limit is to be taken as the RPM corresponding to MCR and the lower limit is to be taken as the RPM corresponding to 75% of MCR.

The natural frequencies of the structure (longitudinal, transverse and torsional) for both dry and wet conditions, in the range of 0 Hz to about 20% above propeller blade frequency, are to be compared against the exciting frequencies.

The natural frequencies are to be outside the band +/-20% of the excitation frequencies.

To ensure that resonance will not occur, it is advisable to arrange that the highest fundamental natural frequency is below the lowest excitation frequency.

In cases where the main propulsion is slow speed diesel engines, a resonance avoidance procedure may not be feasible due to there being several engine excitation frequencies. It is recommended that a Pump Tower model be included in the global ship finite element model for vibration response analysis. This is to be located in the tank adjacent to the engine room. A vibration limit of 15 mm/s (peak) on the Pump Tower is considered to be appropriate.

### Section 4: Re-analysis

Further analyses will generally be required to check the effect of any proposed action on the natural frequencies of the Pump Tower.

Attention is also to be given to the effect of any proposed action on results of the strength and fatigue analyses. In principle, the strength and fatigue checks should be re-evaluated with any modifications introduced as a consequence of the Vibration Check incorporated.

### References:

1. Lloyd's Register’s ShipRight SDA procedure for Sloshing Loads and Scantling Assessment.
2. RP 2A: Planning, Designing, and Constructing Fixed Offshore Platforms
Appendix I:


This extract includes the title pages, contents list and Appendix I ‘Parametric equations used in this study from the original document.

Note: the attached document varies from the original document in that it incorporates a number of editorial corrections to the equations presented.
STRESS CONCENTRATION FACTORS FOR SIMPLE TUBULAR JOINTS
Assessment of Existing and Development of New Parametric Formulae

Prepared by Lloyd’s Register of Shipping for the Health and Safety Executive
STRESS CONCENTRATION FACTORS FOR SIMPLE TUBULAR JOINTS

Assessment of Existing and Development of New Parametric Formulae

Prepared by

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HSE BOOKS

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Results, including detailed evaluation and, where relevant, recommendations stemming from their research projects are published in the OTH/OTI series of reports.
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EXECUTIVE SUMMARY

This report covers the development of a new set of SCF parametric formulae for simple tubular joints, ie the Lloyd’s Register equations, the work being largely funded by the HSE. Additionally, the report covers as assessment of the commonly used SCF equations for simple tubular joints including the new LR equations. This latter work was carried out under the auspices of the HSE Review Panel for Fatigue Guidance (RPFG).

In the development of the LR equations, a comprehensive database of measured SCFs for full-scale steel joints and acrylic models was created. The joint acceptance criteria for this database was agreed with representatives from the Industry. From this database the new LR equations, which are given in Appendix A, were developed.

For the assessment of SCF parametric formulae for simple tubular joints the database above was refined. The finalised database, which is given in Appendix B, was used to assess the existing commonly used parametric formulae and the new LR equations. The assessment criteria which was agreed with the RPFG is given in Section 5.2 and the equations assessed are given in Appendix A.¹

¹ See footnote to Section 1.0 Introduction
NOMENCLATURE: UNSTIFFENED TUBULAR JOINT

INDEX

- D - Chord diameter
- T - Chord thickness
- L - Chord length
- d - Brace diameter
- t - Brace thickness
- θ - Brace to chord inclination
- ∇ - Brace weld toe separation (K & KT Joints)
- C - Chord end fixity condition

SCF - Stress Concentration Factor - Ratio of stress to nominal brace stress
(n.b. For bending, SCF's are relative to the extreme fibre stress)

SCF_{Cs} - SCF at the chord saddle
SCF_{Cc} - SCF at the chord crown
SCF_{C} - Maximum SCF on the chordside
SCF_{Sm} - SCF at the brace saddle
SCF_{Ss} - SCF at the brace crown
SCF_{S} - Maximum SCF on the braceside
APPENDIX A
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A1. LLOYD’S REGISTER PARAMETRIC EQUATIONS

A1.1 Derivation of Equations and Safety Factors

These equations have been derived from the Lloyd’s Register (LR) simple joint SCF database using a least squares minimisation procedure. The quoted equations give an approximately mean fit to the database, however characteristic equations can be derived by applying an appropriate safety factor.

Beside each equation is the percentage standard deviation of the least squares fit to the database ($\sigma\%$), which can be utilised to calculate the required safety factor. It has been found that the LR equations are lognormally distributed about the mean fit line. Therefore, the design curve weighting may be calculated to give a estimated degree of joint underpredictions.

$$SCF \text{ (design)} = SCF \text{ (mean)} \times (1 + n\sigma/100)$$

Where $\eta = \text{Chosen design curve weighting}$

† Exclude the chord in place bending term, $B_0 \times B_1$ and add in expression unfactored.
A1.2  Derived Parametric Equations and Measured SCF Values

The SCF database was standardised to DEn fatigue recommendations prior to curve fitting (ie to results derived from linear extrapolation of maximum principal stresses outside the $0.2\sqrt{(rt)}$ notch zone to the weld toe). Measured SCF results that do not meet this standard should be factored as follows:

- X 0.95 to convert from non-linear extrapolation of stresses
- X 0.95 to simulate a weld fillet on the chordside
- X 0.86 to simulate a weld fillet on the braceside
- X 1.15 to convert perpendicular strain to principal stress (steel models)
- X 1.23 to convert perpendicular strains to principal stress (acrylic models)
A1.3 Lloyd’s Register Equations for T/Y Joints

Notes: When $a<12$ the basic saddle SCF equation should be multiplied by the appropriate short chord correction factor $F_1, F_2$ etc.

Apply the modified $\beta$ value when predicting SCFs at the SADDLE on $\beta = 1$ joints under axial load or OPB.

$%$

Axial load

$SCF_{CS} = T1 \times (F1 \text{ or } F2) \quad \sigma=20\%$

$SCF_{CC} = T2 + B0 \times B1 \quad \sigma=20\%$

$SCF_{BS} = T3 \times (F1 \text{ or } F2) \quad \sigma=25\%$

$SCF_{BC} = T4 \quad \sigma=23\%$

Out-of-plane bending

$SCF_{CS} = T5 \times (F3) \quad \sigma=22\%$

$SCF_{BS} = T6 \times (F3) \quad \sigma=28\%$

In-plane bending

$SCF_{C} = T7 \quad \sigma=15\%$

$SCF_{B} = T8 \quad \sigma=18\%$

Validity range

The above equations for T/Y joints are generally valid for joint parameters within the following limits:

$0.13 \leq \beta \leq 1.0$

$10 \leq \gamma \leq 35$

$0.25 \leq \tau \leq 1.0$

$30^\circ \leq \theta \leq 90^\circ$

$4 \leq a$

Note: $\beta = 1$ joints at the saddle: $\beta = 1 - \left( \frac{x}{y} \times \sin^{0.65} (\Psi^\circ) \right)$

(where $\Psi^\circ$ is the degree of weld cut-back (default value = $20^\circ$)).
A1.4 Lloyd’s Register Equations for X Joints

Notes: When $a < 12$ the basic saddle SCF equation should be multiplied by the appropriate short chord correction factor F1, F2 etc.

Apply the modified $\beta$ value when predicting SCFs at the SADDLE on $\beta = 1$ joints under axial load or OPB.

%  Std  Devn

Balanced axial load

<table>
<thead>
<tr>
<th>SCF</th>
<th>Equation</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{CS}$</td>
<td>$X_1 \times (F_1 \text{ or } F_2)$</td>
<td>22%</td>
</tr>
<tr>
<td>$\text{CC}$</td>
<td>$X_2$</td>
<td>33%</td>
</tr>
<tr>
<td>$\text{BS}$</td>
<td>$X_3 \times (F_1 \text{ or } F_2)$</td>
<td>19%</td>
</tr>
<tr>
<td>$\text{BC}$</td>
<td>$X_4$</td>
<td>13%</td>
</tr>
</tbody>
</table>

Balanced out-of-plane bending

<table>
<thead>
<tr>
<th>SCF</th>
<th>Equation</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{CS}$</td>
<td>$X_5 \times (F_3)$</td>
<td>22%</td>
</tr>
<tr>
<td>$\text{BS}$</td>
<td>$X_6 \times (F_3)$</td>
<td>20%</td>
</tr>
</tbody>
</table>

Balanced in-plane bending

<table>
<thead>
<tr>
<th>SCF</th>
<th>Equation</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{C}$</td>
<td>$X_7$</td>
<td>23%</td>
</tr>
<tr>
<td>$\text{B}$</td>
<td>$X_8$</td>
<td>12%</td>
</tr>
</tbody>
</table>

Validity range

The above equations are generally valid for joint parameters within the following limits:

$0.13 \leq \beta \leq 1.0$
$10 \leq \gamma \leq 35$
$0.25 \leq \tau \leq 1.0$
$30^\circ \leq \theta \leq 90^\circ$
$4 \leq a$

Note: $\beta = 1$ joints at the saddle: $\beta = 1 - \left(\frac{\tau}{\gamma} \times \sin^{0.65}(\Psi^\circ)\right)$

(where $\Psi^\circ$ is the degree of weld cut-back (default value = 20$^\circ$)).
A1.5  Lloyd’s Register K joint equations

Notes:  When $\alpha<12$ the basic saddle SCF equation should be multiplied by the appropriate short chord correction factor $F1$, $F2$ etc.

Apply the modified $\beta$ value when predicting SCFs at the SADDLE on $\beta = 1$ joints under axial load or OPB.

The expressions should be calculated using the geometry associated with brace A, where brace A is always defined as the brace under consideration.

<table>
<thead>
<tr>
<th></th>
<th>%</th>
<th>Std</th>
<th>Devn</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Single axial load</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SCF\textsubscript{CS}</td>
<td>$T1_A \cdot S1_{AB} \times (F1_A \text{ or } F2_A)$</td>
<td>$\sigma=18%$</td>
<td></td>
</tr>
<tr>
<td>SCF\textsubscript{CC}</td>
<td>$T2_A \cdot S2_{AB} + B0_A \times B1_A$</td>
<td>$\sigma=13%$</td>
<td></td>
</tr>
<tr>
<td>SCF\textsubscript{BS}</td>
<td>$T3_A \cdot S1_{AB} \times (F1_A \text{ or } F2_A)$</td>
<td>$\sigma=20%$</td>
<td></td>
</tr>
<tr>
<td>SCF\textsubscript{BC}</td>
<td>$T4_A \cdot S2_{AB}$</td>
<td>$\sigma=23%$</td>
<td></td>
</tr>
<tr>
<td><strong>Balanced axial load</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SCF\textsubscript{CS}</td>
<td>$(T1_A \cdot S1_{AB} - T1_B \cdot S1_{BA} \cdot IF1_{AB}) \times (F1_A \text{ or } F2_A)$</td>
<td>$\sigma=22%$</td>
<td></td>
</tr>
<tr>
<td>SCF\textsubscript{CC}</td>
<td>$(T2_A \cdot S2_{AB} - T2_B \cdot S2_{BA} \cdot IF2_{AB}) + B0_A \times B1_A$</td>
<td>$\sigma=25%$</td>
<td></td>
</tr>
<tr>
<td>SCF\textsubscript{BS}</td>
<td>$(T3_A \cdot S1_{AB} - T3_B \cdot S1_{BA} \cdot IF3_{AB}) \times (F1_A \text{ or } F2_A)$</td>
<td>$\sigma=12%$</td>
<td></td>
</tr>
<tr>
<td>SCF\textsubscript{BC}</td>
<td>$T4_A \cdot S2_{AB} - T4_B \cdot S2_{BA} \cdot IF4_{AB}$</td>
<td>$\sigma=26%$</td>
<td></td>
</tr>
<tr>
<td><strong>Single out-of-plane bending</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SCF\textsubscript{CS}</td>
<td>$T5_A \cdot S1_{AB} \times (F3_A)$</td>
<td>$\sigma=17%$</td>
<td></td>
</tr>
<tr>
<td>SCF\textsubscript{BS}</td>
<td>$T6_A \cdot S1_{AB} \times (F3_A)$</td>
<td>$\sigma=18%$</td>
<td></td>
</tr>
<tr>
<td><strong>Unbalanced out-of-plane bending</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SCF\textsubscript{CS}</td>
<td>$(T5_A \cdot S1_{AB} + T5_B \cdot S1_{BA} \cdot IF5_{AB}) \times (F3_A)$</td>
<td>$\sigma=14%$</td>
<td></td>
</tr>
<tr>
<td>SCF\textsubscript{BS}</td>
<td>$(T6_A \cdot S1_{AB} + T6_B \cdot S1_{BA} \cdot IF6_{AB}) \times (F3_A)$</td>
<td>$\sigma=21%$</td>
<td></td>
</tr>
<tr>
<td><strong>Single in-plane bending</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SCF\textsubscript{C}</td>
<td>$T7_A$</td>
<td>$\sigma=15%$</td>
<td></td>
</tr>
<tr>
<td>SCF\textsubscript{B}</td>
<td>$T8_A$</td>
<td>$\sigma=17%$</td>
<td></td>
</tr>
</tbody>
</table>
Balanced in-plane bending

\[ SCF_C = T7_A + T7_B \text{ IF7}_{AB} \quad \sigma = 15\% \]
\[ SCF_B = T8_A + T8_B \text{ IF8}_{AB} \quad \sigma = 16\% \]

Validity range

The above equations are generally valid for joint parameters within the following limits:

\[
0.13 \leq \beta \leq 1.0 \\
10 \leq \gamma \leq 35 \\
0.25 \leq \tau \leq 1.0 \\
30\degree \leq \theta \leq 90\degree \\
4 \leq \alpha \\
0 \leq \zeta \leq 1.0
\]

Note: \( \beta = 1 \) joints at the saddle: \( \beta = 1 - (\frac{c}{T} \times \sin^{0.65} (\Psi^o)) \)

(where \( \Psi^o \) is the degree of weld cut-back (default value = 20\degree)).
A1.6 Lloyd’s Register KT joint equations

Notes: When $a < 12$ the basic saddle SCF equation should be multiplied by the appropriate short chord correction factor $F1$, $F2$, etc.

Apply the modified $\beta$ value when predicting SCFs at the SADDLE on $\beta = 1$ joints under axial load or OPB.

The expressions should be calculated using the geometry associated with brace A, where brace A is always defined as the brace under consideration, except where otherwise stated.

### Single axial load

$SCF_{CS} = T1_A S1_{AB} S1_{AC} x (F1_A \text{ or } F2_A)$ 

$SCF_{CC} = T2_A S2_{AB} + B0_A x B1_A$  

$SCF_{BC} = T2_B \text{ Max } (S2_{BA}, S2_{BC}) + B0_B x B1_B$  

$SCF_{BS} = T3_A S1_{AB} S1_{AC} x (F1_A \text{ or } F2_A)$  

$SCF_{BC} = T4_A S2_{AB}$  

$SCF_{BC} = T4_B \text{ Max } (S2_{BA}, S2_{BC})$  

### Balanced axial load (only outer braces A and C loaded)

#### Central brace (brace B)

$SCF_{CS} = \text{MAX } [(T1_B S1_{BA} S1_{BC} - T1_A S1_{AB} S1_{AC} IF1_{BA}), (T1_B S1_{BC} S1_{BA} - T1_C S1_{CB} S1_{CA} IF1_{BC})] x (F1_B \text{ or } F2_B)$  

$SCF_{CC} = \text{MAX } [(T2_B S2_{BA} - T2_A S2_{A} IF2_{BA}) + B0_B x B1_B, (T2_B S2_{BA} - T2_C S2_{CB} IF2_{BC}) + B0_B x B1_B]$  

$SCF_{BS} = \text{MAX } [(T3_B S1_{BA} S1_{BC} - T3_A S1_{AB} S1_{AC} IF3_{BA}), T3_B S1_{BC} S1_{BA} - T3_C S1_{CB} S1_{CA} IF3_{BC})] x (F1_B \text{ or } F2_B)$  

$SCF_{BC} = \text{MAX } [(T4_B S2_{BA} - T4_A S2_{A} IF4_{BA}), (T4_B S2_{BA} - T4_C S2_{CB} IF4_{BC})]$  

Where $S2_B = \text{Max } (S2_{BA}, S2_{BC})$, $S2_A = \text{Max } (S2_{AB}, S2_{AC})$ and $S2_C = \text{Max } (S2_{CB}, S2_{CA})$

#### Outer brace (brace A)

$SCF_{CS} = (T1_A S1_{AB} S1_{AC} - T1_C S1_{CB} S1_{CA} IF1_{AC}) x (F1_A \text{ or } F2_A)$  

### Percentages

<table>
<thead>
<tr>
<th>SCF</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SCF_{CS}$</td>
<td>18%</td>
</tr>
<tr>
<td>$SCF_{CC}$</td>
<td>13%</td>
</tr>
<tr>
<td>$SCF_{BC}$</td>
<td>13%</td>
</tr>
<tr>
<td>$SCF_{BS}$</td>
<td>20%</td>
</tr>
<tr>
<td>$SCF_{BC}$</td>
<td>23%</td>
</tr>
<tr>
<td>$SCF_{CS}$</td>
<td>22%</td>
</tr>
<tr>
<td>$SCF_{CC}$</td>
<td>25%</td>
</tr>
<tr>
<td>$SCF_{BS}$</td>
<td>12%</td>
</tr>
<tr>
<td>$SCF_{BC}$</td>
<td>26%</td>
</tr>
<tr>
<td>Outer brace (brace A)</td>
<td>22%</td>
</tr>
</tbody>
</table>
SCF_{CC} = (T_2 A S_2 AB - T_2 c S_2 cb IF_2 AC) + B_0 A x B_1 A \quad \sigma = 25\%

SCF_{BS} = (T_3 A S_1 AB S_1 AC - T_3 c S_1 CB S_1 CA IF_3 AC) \times (F_1 A \text{ or } F_2 A) \quad \sigma = 12\%

SCF_{BC} = (T_4 A S_2 AB - T_4 c S_2 CB IF_4 AC) \quad \sigma = 26\%

**Single out-of-plane bending**

SCF_{CS} = T_5 A S_1 AB S_1 AC \times (F_3 A) \quad \sigma = 17\%

SCF_{BS} = T_6 A S_1 AB S_1 AC \times (F_3 A) \quad \sigma = 18\%

**Unbalanced out-of-plane bending** (all braces loaded)

SCF_{CS} = (T_5 A S_1 AB S_1 AC + T_5 B S_1 BA S_1 BC IF_5 AB
+ T_5 c S_1 CB S_1 CA IF_5 AC) \times (F_3 A) \quad \sigma = 14\%

SCF_{BS} = (T_6 A S_1 AB S_1 AC + T_6 B S_1 BA S_1 BC IF_6 AB
+ T_6 c S_1 CB S_1 CA IF_6 AC) \times (F_3 A) \quad \sigma = 21\%

**Single in-plane bending**

SCF_{C} = T_7 A \quad \sigma = 15\%

SCF_{B} = T_8 A \quad \sigma = 17\%

**Balanced in-plane bending** (only outer braces A & C loaded)

**Central brace (brace B)**

SCF_{CS} = \text{MAX} \left[ (T_1 A S_1 BA S_1 BC - T_1 A S_1 AB S_1 AC IF_1 BA),
(T_1 a S_1 BC S_1 BA - T_1 c S_1 CB S_1 CA IF_1 BC) \right] \times (F_1 A \text{ or } F_2 A) \quad \sigma = 22\%

SCF_{CC} = \text{MAX} \left[ (T_2 A S_2 BA - T_2 A S_2 AB S_2 AC IF_2 BA) + B_0 A x B_1 A,
(T_2 B S_2 BC - T_2 c S_2 CB IF_2 BC) \right] \times (F_1 A \text{ or } F_2 A) \quad \sigma = 25\%

SCF_{BS} = \text{MAX} \left[ (T_3 A S_3 BA S_3 BC - T_3 A S_3 AB S_3 AC IF_3 BA),
(T_3 B S_3 BC S_3 BA - T_3 c S_3 CB S_3 CA IF_3 BC) \right] \times (F_1 A \text{ or } F_2 A) \quad \sigma = 12\%

SCF_{BS} = \text{MAX} \left[ (T_4 A S_4 BA S_4 BC, (T_4 a S_4 BA - T_4 c S_4 CB IF_4 BA)),
(T_4 A S_4 BA S_4 BC, (T_4 a S_4 BA - T_4 c S_4 CB IF_4 BC)) \right] \quad \sigma = 26\%

Where S_{2b} = \text{Max} (S_{2BA}, S_{2BC}), S_{2a} = \text{Max} (S_{2AB}, S_{2AC}) \text{ and } S_{2c} = \text{Max} (S_{2CB}, S_{2CA})

**Outer brace (brace A)**

SCF_{CS} = (T_1 A S_1 AB S_1 AC - T_1 c S_1 CB S_1 CA IF_1 AC) \times (F_1 A \text{ or } F_2 A) \quad \sigma = 22\%

SCF_{CC} = (T_2 A S_1 AB - T_2 c S_2 CB IF_2 AC) + B_0 A x B_1 A \quad \sigma = 25\%

SCF_{BS} = (T_3 A S_1 AB S_1 AC - T_3 c S_1 CB S_1 CA IF_3 AC) \times (F_1 A \text{ or } F_2 A) \quad \sigma = 12\%

SCF_{BC} = (T_4 A S_2 AB - T_4 c S_2 CB IF_4 AC) \quad \sigma = 26\%
Single out-of-plane bending
\[ \text{SCF}_{\text{CS}} = T_{7A} S_{1AB} S_{1AC} x (F_{3A}) \quad \sigma = 17\% \]
\[ \text{SCF}_{\text{BS}} = T_{8A} S_{1AB} S_{1AC} x (F_{3A}) \quad \sigma = 18\% \]

Unbalanced out-of-plane bending (all braces loaded)
\[ \text{SCF}_{\text{CS}} = (T_{5A} S_{1AB} S_{1AC} + T_{5B} S_{1BA} S_{1BC} \text{IF}_{5AB} + T_{5C} S_{1CB} S_{1CA} \text{IF}_{5AC}) x (F_{3A}) \quad \sigma = 14\% \]
\[ \text{SCF}_{\text{BS}} = (T_{6A} S_{1AB} S_{1AC} + T_{6B} S_{1BA} S_{1BC} \text{IF}_{6AB} + T_{6C} S_{1CB} S_{1CA} \text{IF}_{6AC}) x (F_{3A}) \quad \sigma = 21\% \]

Single in-plane bending
\[ \text{SCF}_{\text{C}} = T_{7A} \quad \sigma = 15\% \]
\[ \text{SCF}_{\text{B}} = T_{8A} \quad \sigma = 17\% \]

Balanced in-plane bending (only outer braces A& C loaded)
Central brace (brace B)
\[ \text{SCF}_{\text{C}} = \text{MAX} \left( T_{7B} + T_{7A} \text{IF}_{7AB}, (T_{7B} + T_{7C} \text{IF}_{7BC}) \right) \quad \sigma = 15\% \]
\[ \text{SCF}_{\text{B}} = \text{MAX} \left( T_{8B} + T_{8A} \text{IF}_{8AB}, (T_{8B} + T_{8C} \text{IF}_{8BC}) \right) \quad \sigma = 16\% \]

Outer brace (brace A)
\[ \text{SCF}_{\text{C}} = T_{7A} + T_{7C} \text{IF}_{7AC} \quad \sigma = 15\% \]
\[ \text{SCF}_{\text{B}} = T_{8A} + T_{8C} \text{IF}_{8AC} \quad \sigma = 16\% \]

Validity range
The above equations are generally valid for joint parameters within the following limits:
\[
0.13 \leq \beta \leq 1.0 \\
10 \leq \psi \leq 35 \\
0.25 \leq \tau \leq 1.0 \\
30^\circ \leq \phi \leq 90^\circ \\
4 \leq \alpha \\
0 \leq \zeta \leq 1.0
\]

Note: \( \beta = 1 \) joints at the saddle: \( \beta = 1 - \left( \frac{\psi}{7} \times \sin^{0.65}(\Psi^\circ) \right) \)
(Where \( \Psi^\circ \) is the degree of weld cut-back (default value = 20°))
A1.7 Parametric Equations Expressions

A1.7.1 T Factors - T joint factors

Note: Apply the modified $\beta$ value when predicting SCFs at the \textit{SADDLE} on $\beta = 1$ joints under axial load or OPB

**Axial load**

$T1 = \tau \gamma^{1.2} \beta (2.12-2\beta) \sin^2 \theta$

$T2 = \tau \gamma^{0.2} (3.5-2.4\beta) \sin^{0.3} \theta$

$T3 = 1 + \tau^{0.6} \gamma^{1.3} \beta (0.76 - 0.7\beta) \sin^{2.2} \Theta$

$T4 = 2.6\beta^{0.65} \gamma^{0.3-0.58}$

**Out-of-plane bending**

$T5 = \tau \gamma \beta (1.4-\beta^5) \sin^{1.7} \Theta$

$T6 = 1 + \tau^{0.6} \gamma^{1.3} \beta (0.27 - 0.2\beta^5) \sin^{1.7} \Theta$

**In-plane bending**

$T7 = 1.22\tau^{0.8} \beta \gamma^{(1-0.68\beta) \sin^{(1-\beta^5)}}$\Theta$

$T8 = 1 + \tau^{0.2} \gamma \beta (0.26 - 0.21\beta) \sin^{1.5} \Theta$

A1.7.2 X Factors = X joint factors

Note: Apply the modified $\beta$ value when predicting SCFs at the \textit{SADDLE} on $\beta = 1$ joints under axial load or OPB

**Axial load**

$X1 = \tau \beta \gamma^{1.3} (1.46 - 1.4\beta^3) \sin^5 \Theta$

$X2 = (0.36 + 1.9 \tau^{0.5} \exp^{(\beta^{1.5} \gamma^{0.5})}) (\sin \Theta + 3 \cos^2 \Theta)$

$X3 = 1 + 0.6 \times X1$

$X4 = (1.3 + 0.06 \tau \gamma \exp^{(\beta^{2} \gamma^{0.5})}) \sin -1 \Theta$

**Out-of-plane bending**

$X5 = \tau \beta \gamma^{1.3} (0.63 - 0.6\beta^3) \sin^2 \Theta$

$X6 = 1 + \tau \beta \gamma^{1.3} (0.19 - 0.185\beta^3) \sin^{7(1-\beta^2)} \Theta$

**In-plane bending**

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X7 = τ^{0.8} βγ^{0.32-0.5} \sin^{0.5} Θ

X8 = 1 + τ^{0.8} βγ (0.32-0.25β) \sin^{1.5} Θ

A1.7.3 S Factors - The stiffening effect of an additional brace

Note: Apply the modified β value when predicting SCFs at the SADDLE on β = 1 joints under axial load or OPB (ie Eqn S1_i)

\[ S1_{ij} = \left[ 1 - 0.4 x \exp \left(-30 x_i^2 x \left( \frac{\sin \theta_i}{\sin \theta_j} \right)^2 \right) \right] \]

\[ S2_{ij} = \left[ 1 + \exp \left(-2 x_i^2 x \sin \left(-2 \left( \theta_j \chi_i^\theta \right)^{-0.5} \right) \right) \right] \]

Where \( x_i = 1 + \frac{\zeta_{ij} \sin \theta_i}{\beta_i} \)

\( \zeta_{ij} = \) Gap between weld toes of brace I and brace j/chord diameter

A1.7.4 IF Factors - Influence functions for K and KT joint expressions

Note: Apply the modified β value when predicting SCFs at the SADDLE on β = 1 joints under axial load or OPB

Axial load

\[ IF1_{ij} = \beta_i (2.13-2\beta_i) \gamma^{1.2} \sin \Theta_i \left( \frac{\sin \theta_i}{\sin \theta_j} \right)^P \exp (-0.3 x_i) \text{ where } \]

\( P = 1 \text{ if } \Theta_i > \Theta_j \)

\( P = 5 \text{ if } \Theta_i < \Theta_j \)

\[ IF2_{ij} = \left[ 20-8(\beta_i+1)^2 \right] \exp (-3 x_i) \]

\[ IF3_{ij} = \beta_i (2-1.8\beta_i) \gamma^{1.2} \left( \frac{\sin \theta_i}{\sin \theta_j} \right)^P \exp (-0.5 x_i) \text{ where } \]

\( P = 2 \text{ if } \Theta_i > \Theta_j \)

\( P = 4 \text{ if } \Theta_i < \Theta_j \)

Out-of-plane bending

\[ IF5_{ij} = 0.6 \gamma \left( \frac{\sin \theta_i}{\sin \theta_j} \right) \exp (-3 x_i) \]

\[ IF6_{ij} = 0.14 \beta_i \gamma^{1.5} \left( \frac{\sin \theta_i}{\sin \theta_j} \right) \exp (-3 x_i) \]

In-plane bending

\[ IF7_{ij} = 1.5 \tau_i^{-2} \exp (-3 x_i) \]

\[ IF8_{ij} = [40(\beta_i - 0.75)^2 - 2.5] \exp (-3 x_i) \]

Where \( x_i = 1 + \frac{\zeta_{ij} \sin \theta_i}{\beta_i} \)
\( \zeta = \text{Gap between weld toes of brace } I \text{ and brace } j/\text{chord diameter} \)

**A1.7.5 B Factors - Approximation of the chord in-plane bending**

\[
B_0 = \frac{Ct(B - \tau/2\gamma)(\alpha/2 - \beta/\sin \theta) \sin \theta}{(1 - \tau/2\gamma)} : \text{for single axial load}
\]

\[
B_0 = 0.00 : \text{for balanced axial load}
\]

\[
B_1 = 1.05 + \frac{30 \tau^{1.5} (1.2 - \beta)(\cos^4 \theta + 0.15)}{\gamma}
\]

Chord-end fixity parameter (C) \( 0.5 \leq C \leq 1.0 \)

\( C = 0.5 \) for fully fixed chord ends

\( C = 1.0 \) for pinned chord ends

For a structural analysis a value of \( C = 0.7 \) is normally assumed.

**A1.7.6 F Factors - Short chord correction factors**

Note: Apply the modified \( \beta \) value when predicting SCFs on \( \beta = 1 \) joints

\[
F_1 = \begin{cases} 
1 - (0.83\beta - 0.56\beta^2 - 0.02) \gamma^{0.23} \exp(-0.21\gamma^{-1.16} \alpha^{2.5}) & \alpha < 12 \\
1.0 & \alpha \geq 12
\end{cases}
\]

\[
F_2 = \begin{cases} 
1 - (1.43\beta - 0.97\beta^2 - 0.03) \gamma^{0.04} \exp(-0.71\gamma^{-1.38} \alpha^{2.5}) & \alpha < 12 \\
1.0 & \alpha \geq 12
\end{cases}
\]

\[
F_3 = \begin{cases} 
1 - (0.55\beta^{1.8} \gamma^{0.16} \exp(-0.49\gamma^{-0.89} \alpha^{1.8}) & \alpha < 12 \\
1.0 & \alpha \geq 12
\end{cases}
\]
A2. EFTHYMIOU PARAMETRIC EQUATIONS

A2.1 Efthymiou Equations for T/U Joints

Note: When $\alpha < 12$ the basic saddle SCF equation should be multiplied by the appropriate short chord correction factor $F_1$, $F_2$ etc.

<table>
<thead>
<tr>
<th>Axial load</th>
<th>Short Cord Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SCF_{CS} = \gamma^1 {1.11 - 3(\beta - 0.52)^2} \sin^{1.6} \Theta$ + $(2C - 1)(0.8\alpha - 6) \tau \beta(1 - \beta^2)^{0.5} \sin^3 (2\Theta)$</td>
<td>F1 or F2</td>
</tr>
<tr>
<td>$SCF_{CC} = \gamma^0.2 \tau {2.65 + 5(\beta-0.65)^2} + \tau \beta(0.5C \alpha-3) \sin \Theta$</td>
<td>F1 or F2</td>
</tr>
<tr>
<td>$SCF_{BS} = 1.3 + \gamma \tau^{0.52} \alpha^{0.1} {0.187-1.25\beta^{1.1}(\beta-0.96)} \sin^{2(2.7-0.010)} \Theta$</td>
<td>F1 or F2</td>
</tr>
<tr>
<td>$SCF_{BC} = 3 + \gamma^1.2 {0.12 \exp(-4\beta) + 0.011\beta^2-0.045} + \tau \beta (0.2C \alpha-1.2)$</td>
<td></td>
</tr>
</tbody>
</table>

Out-of-plane bending

| $SCF_{CS} = \gamma \tau \beta(1.7 - 1.05\beta^3) \sin^{1.6} \Theta$ | |
| $SCF_{BS} = \tau^{(0.54)} \gamma^{(-0.05)} \{0.99 - 0.47\beta + 0.08\beta^4\} SCF_{CS}$ | |

In-plane bending

| $SCF_{CC} = 1.45 \beta \tau^{0.85} \gamma^{(1.0-0.68\beta)} \sin^{0.7} \Theta$ | |
| $SCF_{BC} = 1 + 0.65 \beta \tau^{0.4} \gamma^{(1.09-0.77\beta)} \sin^{(0.08\gamma-1.16)} \Theta$ | |

Chord-end fixity parameter (C) $0.5 \leq C \leq 1.0$

$C = 0.5$ for fully fixed chord ends
$C = 1.0$ for pinned chord ends

For a structural analysis a value of $C = 0.7$ is normally assumed.

Short cord correction factors ($\alpha < 12$)

$F1 = 1 - (0.83\beta - 0.56\beta^2 - 0.02)\gamma^{0.23} \exp(-0.21\gamma^{(1.16)} \alpha^{2.5})$
$F2 = 1 - (1.43\beta - 0.97\beta^2 - 0.03)\gamma^{0.04} \exp(-0.71\gamma^{(1.30)} \alpha^{2.5})$
$F3 = 1 - 0.55\beta \gamma^{0.16} \exp(-0.49\gamma^{(-0.89)} \alpha^{1.9})$

Under axial load, apply expression $F1$ if $C < 0.7$ and apply expression $F2$ if $C \geq 0.7$
Validity range

The above equations for T/Y joints are generally valid for joint parameters within the following limits:

\[
\begin{align*}
0.2 & \leq \beta & \leq 1.0 \\
8 & \leq \gamma & \leq 32 \\
0.2 & \leq \tau & \leq 1.0 \\
20^\circ & \leq \Theta & \leq 90^\circ \\
4 & \leq \alpha & \leq 40
\end{align*}
\]
A2.2 Efthymiou Equations for X Joints

Note: When $\alpha < 12$ the basic saddle SCF equation should be multiplied by the appropriate short chord correction factor $F_1$, $F_2$ etc

Axial load

$SCF_{CS} = 3.87 \gamma \tau \beta (1.10 - \beta^{1.8}) \sin^{1.7} \Theta$

$SCF_{CC} = \gamma^{0.2} \tau \{2.65 + 5(\beta - 0.65)^2\} - 3\tau \beta \sin \Theta$

$SCF_{BS} = 1 + 1.9 \gamma \tau^{0.5} \beta^{0.9}(1.09 - \beta^{1.7}) \sin^{2.5} \Theta$

$SCF_{BC} = 3 + \gamma^{1.2} \{0.12 \exp(-4\beta) + 0.011\beta^2 - 0.045\}$

Balanced out-of-plane bending

$SCF_{CS} = \gamma \tau \beta(1.56 - 1.34\beta^4) \sin^{1.6} \Theta$

$SCF_{BS} = \tau^{(0.54)} \gamma^{(0.05)} (0.99 - 0.47\beta + 0.08\beta^4) SCF_{CS}$

Balanced in-plane bending

$SCF_{CC} = 1.45 \beta \tau^{0.85} \gamma^{(1.06\beta)} \sin^{0.7} \Theta$

$SCF_{BC} = 1 + 0.65 \beta \tau^{0.4} \gamma^{(0.99 - 0.77\beta)} \sin^{(0.06\Theta - 1.16)} \Theta$

Chord-end fixity parameter (C) $0.5 \leq C \leq 1.0$

C = 0.5 for fully fixed chord ends
C = 1.0 for pinned chord ends

For a structural analysis a value of $C = 0.7$ is normally assumed.

Short cord correction factors ($\alpha < 12$)

$F_1 = 1 - (0.83\beta - 0.56\beta^2 - 0.02)\gamma^{0.23} \exp(-0.21\gamma^{1.16}\alpha^{2.5})$

$F_2 = 1 - (1.43\beta - 0.97\beta^2 - 0.03)\gamma^{0.04} \exp(-0.71\gamma^{1.38}\alpha^{2.5})$

$F_3 = 1 - 0.55\beta^{1.8}\gamma^{0.16} \exp(-0.49\gamma^{0.89}\alpha^{1.3})$

Under axial load, apply expression $F_1$ if $C < 0.7$ and apply expression $F_2$ if $C \geq 0.7$

Validity range

The above equations for X joints are generally valid for joint parameters within the following limits:

$0.2 \leq \beta \leq 1.0$

$8 \leq \gamma \leq 32$

$0.2 \leq \tau \leq 1.0$

$20^\circ \leq \Theta \leq 90^\circ$

$4 \leq \alpha \leq 40$
A2.3 Efthymiou Equations for K Joints

Note: When $\alpha<12$ the basic saddle SCF equation should be multiplied by the appropriate short chord correction factor $F_1$, $F_2$ etc.

The expressions should be calculated using the geometry associated with brace $A$, where brace $A$ is always defined as the brace under consideration.

For $K$ joint equations, the gap parameter, $x = 1 + z \sin \theta_A / \beta_A$.

**Single axial load**

$$SCF_{CS} = \gamma \tau_A^{1.1} \left\{ 1.11 - 3(\beta_A - 0.52)^2 \right\} \sin^{1.6} \Theta + (2C - 1)(0.8 \alpha - 6) \tau_A \beta_A^2 (1 - \beta_A^2)^{0.5} \sin^2 (2 \Theta_A)$$

$$SCF_{CC} = \gamma \tau_A \{ 2.65 + 5(\beta_A - 0.65)^2 \} + \tau_A \beta_A (0.5C \alpha - 3) \sin \Theta_A$$

$$SCF_{BS} = 1.3 + \gamma \tau_A^{0.52} \alpha^{0.1} \{ 0.187 - 1.25 \beta_A^{1.1} (\beta_A - 0.96) \} \sin^{(2.7 - 0.01 \alpha)} \Theta_A$$

**Balanced axial load**

$$SCF_C = \tau_A^{0.9} \gamma^{0.5} \{ 0.67 - \beta_A^2 + 1.16 \beta_A \} \sin \Theta_A \left[ \frac{\sin \theta_{\text{max}}}{\sin \theta_{\text{min}}} \right]^{0.3} \left[ \frac{\beta_{\text{max}}}{\beta_{\text{min}}} \right]^{0.3} x$$

$$SCF_B = 1 + [SCF_C] (1.97 - 1.57 \beta_A^{0.25}) \tau_A^{0.14} \sin^{0.7} \Theta_A$$  (ATAN in radians)

**Single out-of-plane bending**

$$SCF_{CS} = [\gamma \tau_A \beta_A (1.7 - 1.05 \beta_A^3)] \sin^{1.6} \Theta_A \left[ 1 - 0.08(\beta_A \gamma)^{0.5} \exp(-0.8x) \right]$$  F3

$$SCF_{BS} = \tau_A^{-0.54} \gamma^{-0.05} (0.99 - 0.47 \beta_A + 0.08 \beta_A^3) SCF_{CS}$$  F3

**Unbalanced out-of-plane bending**

$$SCF_{CS} = [\gamma \tau_A \beta_A (1.7 - 1.05 \beta_A^3) \sin^{1.6} \Theta_A \left[ 1 - 0.08(\beta_A \gamma)^{0.5} \exp(-0.8x) \right] + [\gamma \tau_A \beta_B (1.7 - 1.05 \beta_B^3) \sin^{1.6} \Theta_B \left[ 1 - 0.08(\beta_B \gamma)^{0.5} \exp(-0.8x) \right] x [2.05 \beta_{\text{max}}^0.5 \exp(-1.3x)]$$  F4

$$SCF_{BS} = \tau_A^{-0.54} \gamma^{-0.05} (0.99 - 0.47 \beta_A + 0.08 \beta_A^3) \{ SCF_{CS} \}$$  F4
Single in-plane bending

\( SCF_{CC} = 1.45 \beta_A \tau_A^{0.85} \gamma^{1.4 - 0.8\beta} \sin^{0.7} \Theta_A \)

\( SCF_{BC} = 1 + 0.65 \beta_A \tau_A^{0.4} \gamma^{1.09 - 0.77\beta} \sin(0.06\gamma + 1.16) \Theta_A \)

Balanced in-plane bending

\( SCF_{CC} = [1.45 \beta_A \tau_A^{0.85} \gamma^{1.4 - 0.8\beta} \sin^{0.7} \Theta_A][1 + 0.46 \beta_A^{1.2} \exp(-3\zeta)] \)

\( SCF_{BC} = 1 + 0.65 \beta_A \tau_A^{0.4} \gamma^{1.09 - 0.77\beta} \sin(0.06\gamma + 1.16) \Theta_A \)

Chord-end fixity parameter (C) \( 0.5 \leq C \leq 1.0 \)

C = 0.5 for fully fixed chord ends
C = 1.0 for pinned chord ends
For a structural analysis a value of C = 0.7 is normally assumed.

Short cord correction factors (\( \alpha < 12 \))

\( F1 = 1 - (0.83\beta - 0.56\beta^2 - 0.02)\gamma^{0.23} \exp(-0.21\gamma^{1.16} \alpha^{2.5}) \)

\( F2 = 1 - (1.43\beta - 0.97\beta^2 - 0.03)\gamma^{0.04} \exp(-0.71\gamma^{1.38} \alpha^{2.5}) \)

\( F3 = 1 - 0.55\beta^{1.8}\gamma^{0.16} \exp(-0.49\gamma^{0.89} \alpha^{1.3}) \)

\( F4 = 1 - 107\beta^{1.88}\exp(0.16\gamma^{0.16}) \alpha^{2.4} \)

Under axial load, apply expression F1 if C<0.7 and apply expression F2 if C>0.7

Validity range

The above equations for X joints are generally valid for joint parameters within the following limits:

\[
\begin{array}{cccc}
0.2 & \beta & \gamma & \tau \\
8 & 1 & 1 & 1 \\
0.2 & \tau & \gamma & \tau \\
20^\circ & \Theta & \gamma & \Theta \\
4 & \alpha & \gamma & \alpha \\
0.0 & \zeta & \gamma & \zeta \\
1 & 1 & 1 & 1
\end{array}
\]

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A2.4 Efthymiou Equations for KT Joints

Note: The expressions should be calculated using the geometry associated with the brace under consideration ie Central Brace B or Outer Brace A.

**Balanced axial load**

**Central brace (brace B)**

\[
SCF_\text{C} = \tau_B^{0.9} \gamma^{0.5} (0.67 - \beta_B^{3} + 1.16 \beta_B) \sin \Theta_B \left[ \frac{\sin \theta_{\text{max}}}{\sin \theta_{\text{min}}} \right]^{0.3} \left[ \frac{\beta_{\text{max}}}{\beta_{\text{min}}} \right]^{0.3} \times [1.64 + 0.29 \beta_B^{(0.38)} \text{ATAN}(8 \times \text{MAX}(\zeta_{AB}, \zeta_{BC}))]
\]

\[
SCF_B = 1 + [SCF_\text{C}] (1.97 - 1.57 \beta_B^{0.25}) \tau_B^{(0.14)} \sin^{0.7} \Theta_B
\]

\[
(\text{ATAN in radians})
\]

**Outer brace (brace A)**

\[
SCF_\text{C} = \tau_A^{0.9} \gamma^{0.5} (0.67 - \beta_A^{3} + 1.16 \beta_A) \sin \Theta_A \left[ \frac{\text{Max} (\sin \theta_A, \sin \theta_C)}{\text{Min} (\sin \theta_A, \sin \theta_C)} \right]^{0.3} \left[ \frac{\text{Max} (\beta_A, \beta_C)}{\text{Min} (\beta_A, \beta_C)} \right]^{0.3} \times [1.64 + 0.29 \beta_A^{(0.38)} \text{ATAN}(8 \zeta_{AC})]
\]

\[
SCF_A = 1 + [SCF_\text{C}] (1.97 - 1.57 \beta_A^{0.25}) \tau_A^{(0.14)} \sin^{0.7} \Theta_A
\]

Where \( \zeta_{AC} = \zeta_{AB} + \zeta_{BC} + \frac{\beta_B}{\sin \theta_B} \) (ATAN in radians)

**Unbalanced out-of-plane bending**

**Central brace (brace B)**

\[
SCF_\text{C} = \{[\gamma \tau_B \beta_B (1.87 - 1.05 \beta_B^{3}) \sin^{1.6} \Theta_B][1 - 0.08 (\beta_A^{3})^{0.5} \exp(-0.8 \times \text{AB})][\beta_B^{(0.25)} \tau_B^{0.9} \sin^{0.5} (0.67 - \beta_B^{3} + 1.16 \beta_B)] \}
\]

\[
\{1 - 0.08 (\beta_B^{3})^{0.5} \exp(-0.8 \times \text{AB})][2.05 \beta_B^{0.5} \exp(-1.3 \times \text{AB})]
\]

\[
+ [\gamma \tau_B \beta_B (1.7 - 1.05 \beta_B^{3}) \sin^{1.6} \Theta_B]
\]

\[
[1 - 0.08 (\beta_B^{3})^{0.5} \exp(-0.8 \times \text{AB})][2.05 \beta_B^{0.5} \exp(-1.3 \times \text{AB})]
\]

Where \( x_{\text{AB}} = 1 + \frac{\zeta_{AB} \sin \theta_B}{\beta_B} \) and \( x_{\text{BC}} = 1 + \frac{\zeta_{BC} \sin \theta_B}{\beta_B} \)

\[
SCF_B = \tau_B^{(-0.54)} \gamma^{0.05} (0.99 - 0.47 \beta_B + 0.08 \beta_B^{0.9} \text{SCF}_C)
\]

**Outer brace (brace A)**

\[
SCF_\text{C} = \{[\gamma \tau_B \beta_B (1.7 - 1.05 \beta_B^{3}) \sin^{1.6} \Theta_B][1 - 0.08 (\beta_B^{3})^{0.5} \exp(-0.8 \times \text{AB})][1.0 - 0.08 (\beta_C^{3})^{0.5} \exp(-0.8 \times \text{AC})][1.0 - 0.08 (\beta_C^{3})^{0.5} \exp(-0.8 \times \text{AC})]
\]

\[
+ [\gamma \tau_B \beta_B (1.7 - 1.05 \beta_B^{3}) \sin^{1.6} \Theta_B]
\]

Where \( x_{\text{AB}} = 1 + \frac{\zeta_{AB} \sin \theta_B}{\beta_B} \) and \( x_{\text{BC}} = 1 + \frac{\zeta_{BC} \sin \theta_B}{\beta_B} \)
\[1 - 0.08(\beta_A \gamma)^{0.5} \exp(-0.8x_{AB})][2.05\beta_{\text{max}}^{0.5} \exp(-1.3x_{AB})] + [\gamma_c \beta_c (1.7 - 1.05\beta_c^{1.6}) \sin^{1.6} \Theta_c]\]
\[1 - 0.08(\beta_A \gamma)^{0.5} \exp(-0.8x_{AC})][2.05\beta_{\text{max}}^{0.5} \exp(-1.3x_{AC})]\]

Where \(x_{AB} = 1 + \frac{\zeta_{AB} \sin \theta_A}{\beta_A}\) and \(x_{AC} = 1 + \left(\frac{\zeta_{AB} + \zeta_{BC} + \beta_A \sin \theta_A}{\beta_A} \right) \sin \theta_A\)

\[\text{SCF}_B = \tau_A^{(-0.54)} \gamma^{(-0.05)} (0.99 - 0.47\beta_A + 0.08 \beta_A^4) \text{SCF}_C\]

Balanced in-plane bending
Central brace (brace B)

\[\text{SCF}_C = 1.45 \beta_B \tau_B^{0.85} \gamma^{(1 - 0.68)\beta_B} \sin \Theta_B \left[1 + 0.46\beta_B^{1.2} \exp(-3\min(\zeta_{AB}, \zeta_{BC}))\right]\]

\[\text{SCF}_B = 1 + 0.65\beta_B \tau_B^{0.4} \gamma^{(1.09 - 0.77)\beta_B} \sin^{0.06}\Theta_A\]

Outer brace (brace A)

\[\text{SCF}_C = 1.45 \beta_A \tau_A^{0.85} \gamma^{(1 - 0.68)\beta_A} \sin \Theta_A \left[1 + 0.46\beta_A^{1.2} \exp(-3\zeta_{AB})\right]\]

\[\text{SCF}_B = 1 + 0.65\beta_A \tau_A^{0.4} \gamma^{(1.09 - 0.77)\beta_A} \sin^{0.06}\Theta_A\]

Chord-end fixity parameter (C) \(0.5 \leq C \leq 1.0\)

\[C = 0.5\) for fully fixed chord ends
\[C = 1.0\) for pinned chord ends

For a structural analysis a value of \(C = 0.7\) is normally assumed.

Validity range

The above equations for KT joints are generally valid for joint parameters within the following limits:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta)</td>
<td>0.2</td>
<td>1.0</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>8</td>
<td>32</td>
</tr>
<tr>
<td>(\tau)</td>
<td>0.2</td>
<td>1.0</td>
</tr>
<tr>
<td>(\Theta)</td>
<td>20°</td>
<td>90°</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>4</td>
<td>40</td>
</tr>
<tr>
<td>(\zeta)</td>
<td>0.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>